



NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE
(NAAC Accredited)
(Approved by AICTE, Affiliated to APJ Abdul Kalam Technological University, Kerala)



DEPARTMENT OF MECHANICAL ENGINEERING
COURSE MATERIALS



EST 130 BASICS OF ELECTRICAL ENGINEERING

VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

ABOUT DEPARTMENT

- ◆ Established in: 2002
- ◆ Course offered : B.Tech in Mechanical Engineering
- ◆ Approved by AICTE New Delhi and Accredited by NAAC
- ◆ Affiliated to the University of Dr. A P J Abdul Kalam Technological University.

DEPARTMENT VISION

Producing internationally competitive Mechanical Engineers with social responsibility & sustainable employability through viable strategies as well as competent exposure oriented quality education.

DEPARTMENT MISSION

1. Imparting high impact education by providing conducive teaching learning environment.
2. Fostering effective modes of continuous learning process with moral & ethical values.
3. Enhancing leadership qualities with social commitment, professional attitude, unity, team spirit & communication skill.
4. Introducing the present scenario in research & development through collaborative efforts blended with industry & institution.

PROGRAMME EDUCATIONAL OBJECTIVES

PEO1: Graduates shall have strong practical & technical exposures in the field of Mechanical Engineering & will contribute to the society through innovation & enterprise.

PEO2: Graduates will have the demonstrated ability to analyze, formulate & solve design engineering / thermal engineering / materials & manufacturing / design issues & real life problems.

PEO3: Graduates will be capable of pursuing Mechanical Engineering profession with good communication skills, leadership qualities, team spirit & communication skills.

PEO4: Graduates will sustain an appetite for continuous learning by pursuing higher education & research in the allied areas of technology.

PROGRAM OUTCOMES (POS)

Engineering Graduates will be able to:

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to

provide valid conclusions.

5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSO)

PSO1: graduates able to apply principles of engineering, basic sciences & analytics including multi variant calculus & higher order partial differential equations..

PSO2: Graduates able to perform modeling, analyzing, designing & simulating physical systems, components & processes.

PSO3: Graduates able to work professionally on mechanical systems, thermal systems & production systems.

COURSE OUTCOME

After the completion of the course the student will be able to

CO 1	Apply fundamental concepts and circuit laws to solve simple DC electric circuits.
CO 2	Develop and solve models of magnetic circuits.

CO 3	Apply the fundamental laws of electrical engineering to solve simple ac circuits in steady state.
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CO VS PO'S AND PSO'S MAPPING

CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO 1	3	1	3	-	-	-	-	-	-	-	-	2	2		
CO 2	3	1	2	-	-	-	-	-	-	-	-	2	2		
CO 3	3	1	3	-	-	-	-	-	-	-	-	2	2		

Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1

SYLLABUS

MODULE I

No	Topic	No. of Lectures
1	<i>Elementary Concepts of Electric Circuits</i>	
1.1	Elementary concepts of DC electric circuits: Basic Terminology including voltage, current, power, resistance, emf; Resistances in series and parallel; Current and Voltage Division Rules; Capacitors & Inductors: V-I relations and energy stored. Ohms Law and Kirchhoff's laws-Problems; Star-delta conversion (resistive networks only-derivation not required)- problems.	1 2 1
1.2	Analysis of DC electric circuits: Mesh current method - Matrix representation - Solution of network equations. Node voltage methods-matrix representation-solution of network equations by matrix methods. Numerical problems.	1 1 2

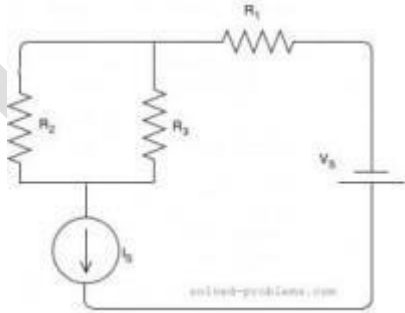
MODULE II

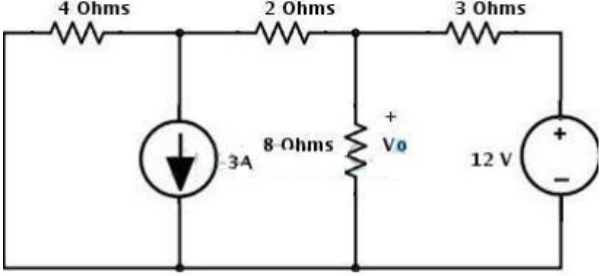
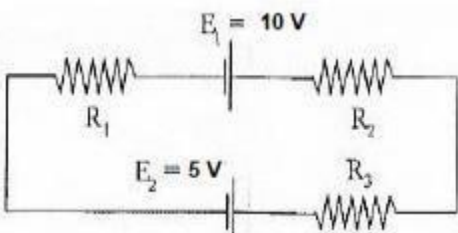
2	Elementary Concepts of Magnetic circuits, Electromagnetic Induction and AC fundamentals	
2.1	Magnetic Circuits: Basic Terminology: MMF, field strength, flux density, reluctance - comparison between electric and magnetic circuits- Series and parallel magnetic circuits with composite materials, numerical problems.	1 2
2.2	Electromagnetic Induction: Faraday's laws, problems, Lenz's law- statically induced and dynamically induced emfs - Self-inductance and mutual inductance, coefficient of coupling	1 2
2.3	Alternating Current fundamentals: Generation of alternating voltages- Representation of sinusoidal waveforms: frequency, period, Average, RMS values and form factor of waveforms-Numerical Problems.	2

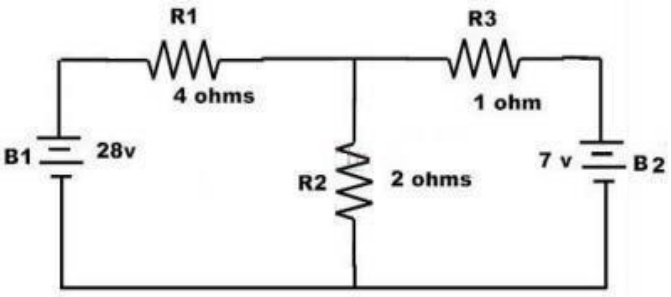
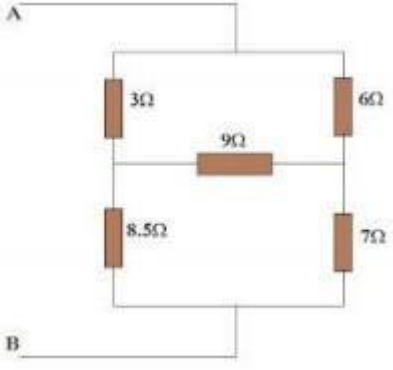
MODULE III

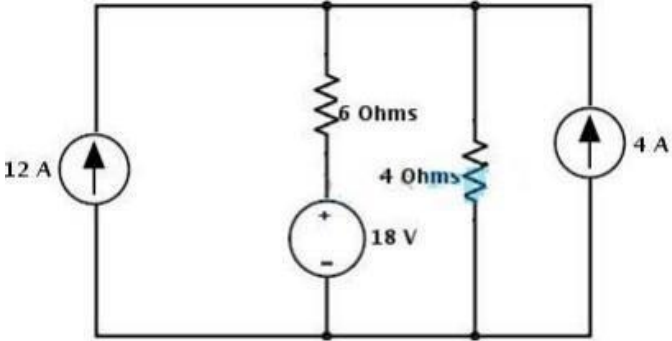
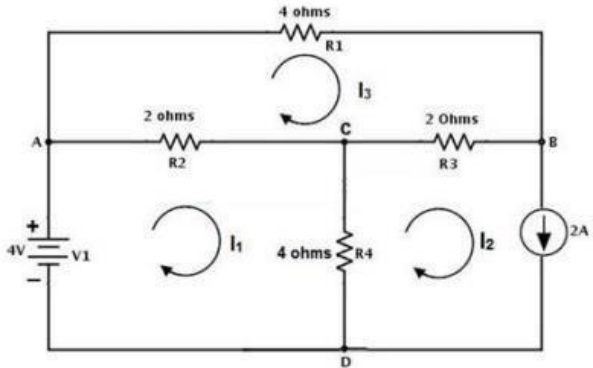
3.1	AC Circuits: Phasor representation of sinusoidal quantities. Trigonometric, Rectangular, Polar and complex forms. Analysis of simple AC circuits: Purely resistive, inductive & capacitive circuits; Inductive and capacitive reactance, concept of impedance. Average Power, Power factor. Analysis of RL, RC and RLC series circuits-active, reactive and apparent power. Simple numerical problems.	1 2 1 2
3.2	Three phase AC systems: Generation of three phase voltages; advantages of three phase systems, star and delta connections (balanced only), relation between line and phase voltages, line and phase currents- Numerical problems.	2

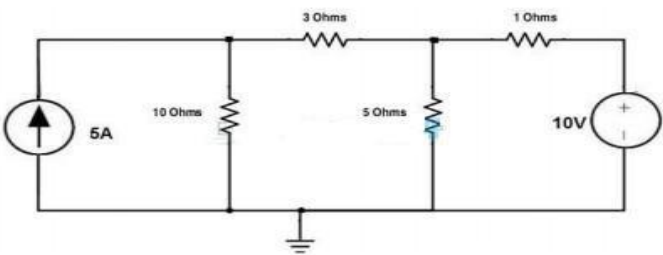
QUESTION BANK

MODULE I				
Q:NO:	QUESTIONS	CO	KL	PAGE NO:
1	Define Electric circuit.	CO1	K1	18
2	Discuss in detail about the various types of electric networks.	CO1	K2	25
3	What are the basic properties of electric circuit?	CO1	K1	26
4	With neat sketch compare unilateral and bilateral circuit.	CO1	K4	27
5	Discuss in detail about the voltage and current relationships in open, closed and short circuit.	CO1	K2	28
6	Compare active and passive electrical components with examples.	CO1	K4	36
7	Elucidate current division rule and voltage division rule with necessary equations.	CO1	K3	36
8	<p>Find current of resistors, use the current division rule.</p> 	CO1	K2	39
9	Discuss in detail about the concept of source transformation with necessary sketches and equations.	CO1	K2	40

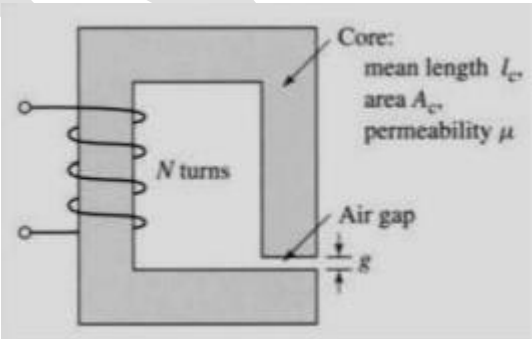
10	Consider the below voltage source circuit with a voltage of 20 V and a internal resistance of 5 ohms. This circuit is transformed into the current source by placing a resistor of the same value with a current source. What will be the value of current source.	CO1	K2	44
11	Apply source transformation to simplify the circuit below. 	CO1	K3	45
12	Interpret the Kirchoff's first and second law with suitable sketches and necessary equations.	CO1	K3	52
13	Define branch, node, loop and mesh.	CO1	K1	53
14	List the various applications of KCL and KVL.	CO1	K1	58
15	If $R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 6\Omega$, determine the electric current that flows in the circuit below. 	CO1	K4	59

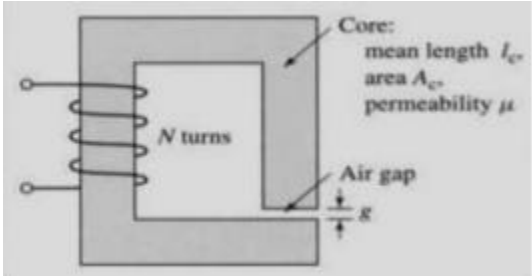
16	<p>Consider the below typical two loop circuit where we have to find the currents I_1 and I_2 by applying the Kirchhoff's laws.</p> 	CO1	K4	62
17	<p>Discuss the star-Delta transformation and write the transforming equations.</p>	CO1	K2	63
18	<p>Find the total resistance between A&B terminals for the network shown in figure.</p> 	CO1	K2	68
19	<p>Discuss in detail about the various steps involved in solving mesh analysis.</p>	CO1	K2	73

20	<p>Consider the below figure, find the voltage across the 12A current source using mesh analysis. In the given circuit all the sources are current sources.</p> 	CO1	K2	75
21	<p>Consider the below circuit where we determine the voltage across the current source and a branch current I_{ac}. Assign the directions as shown below and note that current is assigned opposite to the source current in second loop.</p> 	CO1	K4	77
22	<p>Discuss in detail about the various steps involved in solving node analysis.</p>	CO1	K2	83

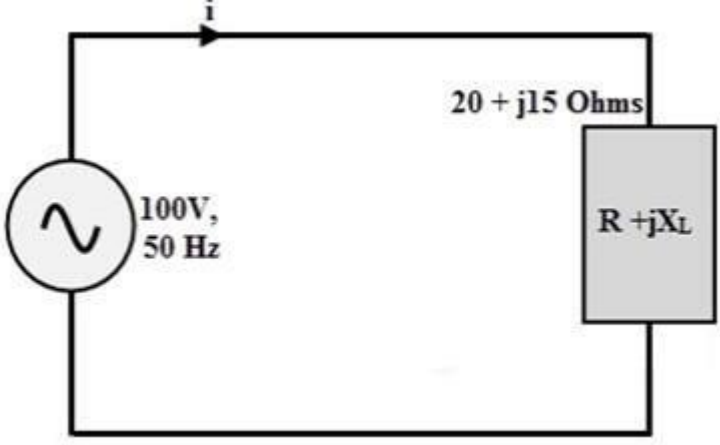
23	<p>Determine the node voltages and currents in each branch using nodal analysis method in the given circuit.</p> 	CO1	K2	89
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MODULE II

1	Define magnetic circuit.	CO2	K1	93
2	Investigate in detail about series and parallel magnetic circuit.	CO2	K6	95
3	Discuss in detail about Magnetic flux, Flux density, Flux intensity, MMF, reluctance and Permeability with necessary equations.	CO2	K2	96
4	Define leakage flux and fringing in magnetic circuits.	CO2	K1	100
5	<p>A magnetic circuit with a single air gap is shown in Fig. The core dimensions are: Cross-sectional area $A_c = 1.8 \times 10^{-3} \text{ m}^2$ Mean core length $l_c = 0.6 \text{ m}$ Gap length $g = 2.3 \times 10^{-3} \text{ m}$ $N = 83$ turns.</p> 	CO2	K4	101

6	<p>Consider the magnetic circuit of with the dimensions of Problem. Assuming infinite core permeability, calculate (a) the number of turns required to achieve an inductance of 12 mH and (b) the inductor current which will result in a core flux density of 1.0 T.</p> 	CO2	K4	103
7	Compare electric circuit's vs magnetic circuit.	CO2	K4	103
8	Define electromagnetic induction and the basic statement behind faradays law of EMI.	CO2	K1	108
9	Discuss the Flemming's motor rule and generator rule with necessary sketches.	CO2	K2	111
10	Investigate in detail about the types of induced emf with necessary sketches and equations.	CO2	K6	116
11	Compare statically induced emf and dynamically induced emf.	CO2	K4	116
12	Discuss in detail about coefficient of self-inductance, mutual inductance and coefficient of coupling.	CO2	K2	120
13	A solenoid of 500 turns is wound on an iron core of relative permeability 800. The length and radius of the solenoid are 40 cm and 3 cm respectively. Calculate the average emf induced in the solenoid if the current in it changes from 0 to 3 A in 0.4 second.	CO2	K4	124
14	The self-inductance of an air-core solenoid is 4.8 mH. If its core is replaced by iron core, then its self-inductance becomes 1.8 H. Find out the relative permeability of iron	CO2	K4	125

15	Compare AC and DC.	CO2	K4	130
16	Discuss in detail about the generation of single phase alternating emf.	CO2	K2	132
17	Define period, frequency, peak value and instantaneous value in a sinusoidal wave form.	CO2	K1	137
18	Derive rms value and average value for sinusoidal wave.	CO2	K4	141
19	A sinusoidal alternating current of 6 amps is flowing through a resistance of 40Ω . Calculate the average voltage and the peak voltage of the supply.	CO2	K3	145
MODULE III				
1	Discuss in detail about the phasor representation of ac.	CO3	K2	149
2	Define phase and phase difference.	CO3	K1	151
3	With suitable sketches discuss in detail about in-phase, lagging phase and leading phase.	CO3	K3	154
4	Discuss in detail about the analysis of purely Resistance circuit.	CO3	K2	157
5	Discuss in detail about the analysis of purely Inductive and Capacitive circuit.	CO3	K2	158
6	Discuss in detail about the analysis of R-L circuit.	CO3	K2	161
7	Discuss in detail about the analysis of R-C circuit.	CO3	K2	163
8	Discuss in detail about the analysis of series R-L-C circuit.	CO3	K2	164
9	Discuss in detail about the analysis of parallel R-L-C circuit.	CO3	K2	172
10	In the circuit shown in Figure the current is 1.8 A. If the current through the capacitor is 1.5 A, find the applied voltage and the resistance of the resistor	CO3	K3	173

11	Discuss in detail about the types of ac power.	CO3	K2	177
12	Define power factor.	CO3	K1	182
13	<p>If an AC power supply of 100V, 50Hz is connected across a load of impedance, $20 + j15$ Ohms. Then calculate the current flowing through the circuit, active power, apparent power, reactive power and power factor.</p> 	CO3	K3	182
14	Discuss in detail about three phase star connected systems and its voltage and current relations.	CO3	K2	185
15	Discuss in detail about three phase Delta connected systems and its voltage and current relations.	CO3	K2	186
16	List the various advantages of three phase systems	CO3	K1	190
17	The input power to a 3-phase a.c. motor is measured as 5kW. If the voltage and current to the motor are 400V and 8.6A respectively, determine the power factor of the system?	CO3	K3	199
18	Three identical coils, each of resistance 10ohm and inductance 42mH are connected (a) in star and (b) in delta to a 415V, 50 Hz, 3-phase supply. Determine the total power dissipated in each case.	CO3	K3	201

APPENDIX 1

CONTENT BEYOND THE SYLLABUS

S:NO;	TOPIC	PAGE NO:
1	DC MACHINES CONSTRUCTION	206
2	DC GENERATORS TYPES AND ITS WORKING.	211

MODULE I

ELEMENTARY CONCEPTS OF ELECTRIC CIRCUIT

INTRODUCTION

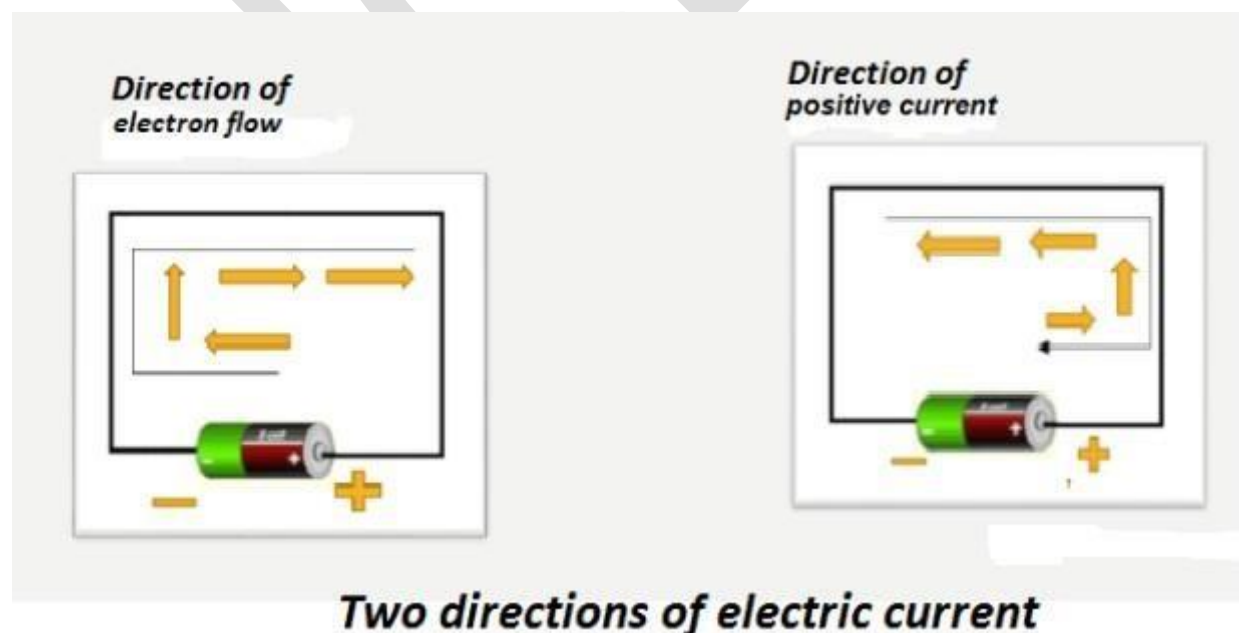
Understanding of electrical terms is compulsory for engineers. As you study electricity in your engineering program, and as you work with electricity in the power plants, industrial plants, you will hear, read, and use various electrical terms. These terms have very exact meanings. You must know what each one means if you are to understand other people and make them understand you. The following terms explain the meaning of the most basic electrical terms.

Electric current the electrical term

Electric current is one of the basic electrical terms described as when electrons flow from one place to another, they make a current. The electrons always flow from a negative point to a positive (or less negative) point, because electrons have a negative charge.

Unfortunately, the direction of the current flow can be confusing. Some people think of a positive current that is in the opposite direction from the electron flow- that is, from positive to negative instead of from negative to positive. You must be careful to distinguish between the two kinds of flow. Both kinds are commonly used in words and diagrams.

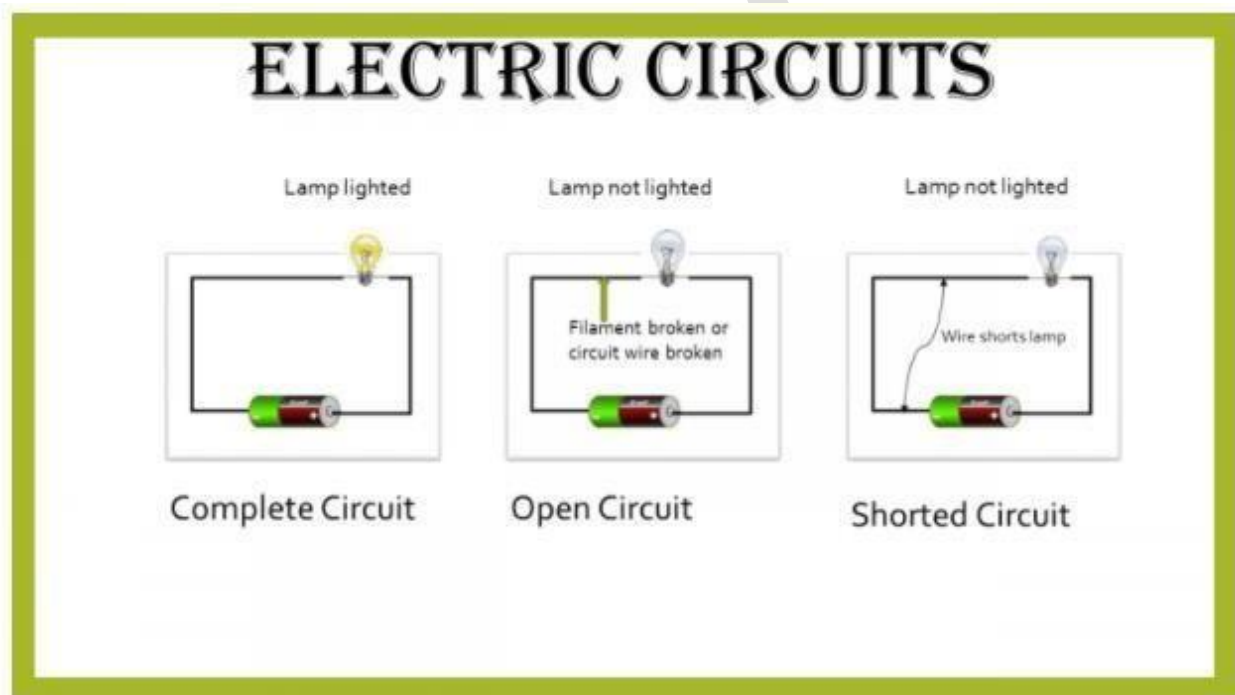
The so-called -positive current| is from positive to negative. The -electron flow| is from negative to positive. The word -current| to mean electron flow-from negative to positive. The figure below shows the difference between positive current and electron flow.



Circuit

Electrons flow along with some kind of path in going from one point to another. This path is called a circuit. If the path has no gaps to stop the flow of electrons, the circuit is said to be complete or closed.

In case the path has a gap that the electrons cannot cross- for example, a break in a wire where the ends are separated by air- the circuit is said to be open. If another pathway provides an easier way a short circuit for the electrons to go from one point another, that part of the circuit is said to be shorted. The figure below shows the circuit that is complete, open, and shorted.



Potential difference

This term is the correct name for what is often called -voltage or -electromotive force. Potential difference is a measure of how much potential energy an electron has in one place compared to another place.

The greater the potential energy, the more work an electron can do in going from one place to the other. The potential energy of each electron also determines how much current will flow from one point to another in a given circuit.

Resistance

Every electrical pathway from one place to another has the property of resisting the flow of electrons. Some pathways resist the flow only slightly. For example, a thick copper wire offers very little resistance. Other pathways—for examples, an air gap—offer great resistance. The greater the resistance, the less the current for a given potential difference.

Cell

Electricity can be produced by chemical means. The arrangement of materials that produces a potential difference between two points by chemical resources is called a cell. Familiar cells include the dry cells used in flashlights, calculators, and radios.

Battery

When you connect two or more cells together, the combination is called a battery. If your calculator takes two dry cells, the grouping is called a two-cell battery.

The storage battery in a car or truck is usually a six-cell battery that produces a potential difference of 12V between the terminals. Each cell in such a battery produces a potential difference of 2 V between its internal terminals. The terminals of the cells are connected in such a way that their potential differences add together between the external terminals

As you study the remaining lessons in this course and the other courses in your series, make sure you learn the proper terms to use in discussing electricity, and the exact meaning of each term. Always use the correct terms, even if other people do not. If you use the wrong terms, you are likely to be misunderstood. In addition, other people will think you know less about electricity than you actually do.

Alternating Current (AC) — An electric current that reverses its direction many times a second at regular intervals.

Ammeter — An instrument for measuring the flow of electrical current in amperes. Ammeters are always connected in series with the circuit to be tested.

Ampacity — The maximum amount of electric current a conductor or device can carry before sustaining immediate or progressive deterioration.

Ampere-Hour (Ah) — A unit of measure for battery capacity. It is obtained by multiplying the current (in amperes) by the time (in hours) during which current flows. For example, a battery which provides 5 amperes for 20 hours is said to deliver 100 ampere – hours.

Ampere (A) — A unit of measure for the intensity of an electric current flowing in a circuit. One ampere is equal to a current flow of one coulomb per second.

Apparent Power — Measured in volt-amperes (VA). Apparent power is the product of the rms

voltage and the rms current.

Armature — The movable part of a generator or motor. It is made up of conductors which rotate through a magnetic field to provide voltage or force by electromagnetic induction. The pivoted points in generator regulators are also called armatures.

Capacitance — The ability of a body to store an electrical charge. Measured in farads as the ratio of the electric charge of the object (Q , measured in coulombs) to the voltage across the object (V , measured in volts).

Capacitor — A device used to store an electric charge, consisting of one or more pairs of conductors separated by an insulator. Commonly used for filtering out voltage spikes.

Circuit — A closed path in which electrons from a voltage or current source flow. Circuits can be in series, parallel, or in any combination of the two.

Circuit Breaker — An automatic device for stopping the flow of current in an electric circuit. To restore service, the circuit breaker must be reset (closed) after correcting the cause of the overload or failure. Circuit breakers are used in conjunction with protective relays to protect circuits from faults.

Conductor — Any material where electric current can flow freely. Conductive materials, such as metals, have a relatively low resistance. Copper and aluminum wire are the most common conductors.

Corona — A corona discharge is an electrical discharge brought on by the ionization of a fluid such as air surrounding a conductor that is electrically charged. Spontaneous corona discharges occur naturally in high-voltage systems unless care is taken to limit the electric field strength.

Current (I) — The flow of an electric charge through a conductor. An electric current can be compared to the flow of water in a pipe. Measured in amperes.

Cycle — The change in an alternating electrical sine wave from zero to a positive peak to zero to a negative peak and back to zero. *See Frequency.*

Demand — The average value of power or related quantity over a specified period of time.

Dielectric constant — A quantity measuring the ability of a substance to store electrical energy in an electric field.

Dielectric strength — The maximum electric field that a pure material can withstand under ideal conditions without breaking down (i.e., without experiencing failure of its insulating properties).

Diode — A semiconductor device with two terminals, typically allowing the flow of current in

one direction only. Diodes allow current to flow when the anode is positive in relation to the cathode.

Direct Current (DC) — An electric current that flows in only one direction.

Electrolyte — Any substance which, in solution, is dissociated into ions and is thus made capable of conducting an electrical current. The sulfuric acid – water solution in a storage battery is an electrolyte.

Electromotive Force — (EMF) A difference in potential that tends to give rise to an electric current. Measured in volts.

Electron — A tiny particle which rotates around the nucleus of an atom. It has a negative charge of electricity.

Electron theory — The theory which explains the nature of electricity and the exchange of free electrons between atoms of a conductor. It is also used as one theory to explain direction of current flow in a circuit.

Farad — A unit of measure for capacitance. One farad is equal to one coulomb per volt.

Ferroresonance — (nonlinear resonance) a type of resonance in electric circuits which occurs when a circuit containing a nonlinear inductance is fed from a source that has series capacitance, and the circuit is subjected to a disturbance such as opening of a switch. It can cause overvoltages and overcurrents in an electrical power system and can pose a risk to transmission and distribution equipment and to operational personnel.

Frequency — The number of cycles per second. Measured in Hertz. If a current completes one cycle per second, then the frequency is 1 Hz; 60 cycles per second equals 60 Hz.

Fuse — A circuit interrupting device consisting of a strip of wire that melts and breaks an electric circuit if the current exceeds a safe level. To restore service, the fuse must be replaced using a similar fuse with the same size and rating after correcting the cause of failure.

Generator — A device which converts mechanical energy into electrical energy.

Ground — The reference point in an electrical circuit from which voltages are measured, a common return path for electric current, or a direct physical connection to the Earth.

Ground Fault Circuit Interrupters (GFCI) — A device intended for the protection of personnel that functions to de-energize a circuit or portion thereof within an established period of time when a current to ground exceeds some predetermined value that is less than that required to operate the overcurrent protective device of the supply circuit.

Henry — A unit of measure for inductance. If the rate of change of current in a circuit is one

ampere per second and the resulting electromotive force is one volt, then the inductance of the circuit is one henry.

Hertz — A unit of measure for frequency. Replacing the earlier term of cycle per second (cps).

Impedance — The measure of the opposition that a circuit presents to a current when a voltage is applied. Impedance extends the concept of resistance to AC circuits, and possesses both magnitude and phase, unlike resistance, which has only magnitude.

Inductance — The property of a conductor by which a change in current flowing through it induces (creates) a voltage (electromotive force) in both the conductor itself (self-inductance) and in any nearby conductors (mutual inductance). Measured in henry (H).

Inductor — A coil of wire wrapped around an iron core. The inductance is directly proportional to the number of turns in the coil.

Insulator — Any material where electric current does not flow freely. Insulative materials, such as glass, rubber, air, and many plastics have a relatively high resistance. Insulators protect equipment and life from electric shock.

Inverter — An apparatus that converts direct current into alternating current.

Kilowatt-hour (kWh) — The product of power in Kw and time in hours. Equal to 1000 Watt-hours. For example, if a 100W light bulb is used for 4 hours, 0.4kWhs of energy will be used (100W x 4 hours / 1000 Watts = 0.4 kWh). Electrical energy is sold in units of kWh.

Kilowatt-hour Meter — A device used to measure electrical energy use.

Kilowatt (Kw) — Equal to 1000 watts.

Load — Anything which consumes electrical energy, such as lights, transformers, heaters and electric motors.

Load Rejection — The condition in which there is a sudden load loss in the system which causes the generating equipment to be over-frequency. A load rejection test confirms that the system can withstand a sudden loss of load and return to normal operating conditions using its governor. Load banks are normally used for these tests as part of the commissioning process for electrical power systems.

Mutual Induction — Occurs when changing current in one coil induces voltage in a second coil.

Ohm — (Ω) A unit of measure of resistance. One ohm is equivalent to the resistance in a circuit transmitting a current of one ampere when subjected to a potential difference of one volt.

Ohm's Law — The mathematical equation that explains the relationship between current, voltage, and resistance ($V=IR$).

Ohmmeter — An instrument for measuring the resistance in ohms of an electrical circuit.

Open Circuit — An open or open circuit occurs when a circuit is broken, such as by a broken wire or open switch, interrupting the flow of current through the circuit. It is analogous to a closed valve in a water system.

Parallel Circuit — A circuit in which there are multiple paths for electricity to flow. Each load connected in a separate path receives the full circuit voltage, and the total circuit current is equal to the sum of the individual branch currents.

Piezoelectricity — Electric polarization in a substance (especially certain crystals) resulting from the application of mechanical stress (pressure).

Polarity — A collective term applied to the positive (+) and negative (-) ends of a magnet or electrical mechanism such as a coil or battery.

Power — The rate at which electrical energy is transferred by an electric circuit. Measured in Watts.

Power Factor — The ratio of the actual electrical power dissipated by an AC circuit to the product of the r.m.s. values of current and voltage. The difference between the two is caused by reactance in the circuit and represents power that does no useful work.

Protective Relay — A relay device designed to trip a circuit breaker when a fault is detected.

Reactive Power — The portion of electricity that establishes and sustains the electric and magnetic fields of AC equipment. Exists in an AC circuit when the current and voltage are not in phase. Measured in VARS.

Rectifier — An electrical device that converts an alternating current into a direct one by allowing a current to flow through it in one direction only.

Relay — An electrical coil switch that uses a small current to control a much larger current.

Reluctance — The resistance that a magnetic circuit offers to lines of force in a magnetic field.

Resistance — The opposition to the passage of an electric current. Electrical resistance can be compared to the friction experienced by water when flowing through a pipe. Measured in ohms.

Resistor — A device usually made of wire or carbon which presents a resistance to current flow.

Rotor — The rotating part of an electrical machine such as a generator, motor, or alternator.

Self Induction — Voltage which occurs in a coil when there is a change of current.

Semiconductor — A solid substance that has a conductivity between that of an insulator and that of most metals, either due to the addition of an impurity or because of temperature effects. Devices made of semiconductors, notably silicon, are essential components of most electronic circuits.

Series-Parallel Circuit — A circuit in which some of the circuit components are connected in series and others are connected in parallel.

Series Circuit — A circuit in which there is only one path for electricity to flow. All of the current in the circuit must flow through all of the loads.

Service — The conductors and equipment used to deliver energy from the electrical supply system to the system being served.

Short Circuit — When one part of an electric circuit comes in contact with another part of the same circuit, diverting the flow of current from its desired path.

Solid State Circuit — Electronic (integrated) circuits which utilize semiconductor devices such as transistors, diodes and silicon controlled rectifiers.

Transistor — A semiconductor device with three connections, capable of amplification in addition to rectification.

True Power — Measured in Watts. The power manifested in tangible form such as electromagnetic radiation, acoustic waves, or mechanical phenomena. In a direct current (DC) circuit, or in an alternating current (AC) circuit whose impedance is a pure resistance, the voltage and current are in phase.

VARs — A unit of measure of reactive power. Vars may be considered as either the imaginary part of apparent power, or the power flowing into a reactive load, where voltage and current are specified in volts and amperes.

Variable Resistor — A resistor that can be adjusted to different ranges of value.

Volt-Ampere (VA) — A unit of measure of apparent power. It is the product of the rms voltage and the rms current.

Volt (V) — A unit measure of voltage. One volt is equal to the difference of potential that would drive one ampere of current against one ohm resistance.

Voltage — An electromotive force or -pressure that causes electrons to flow and can be compared to water pressure which causes water to flow in a pipe. Measured in volts.

Voltmeter — An instrument for measuring the force in volts of an electrical current. This is the difference of potential (voltage) between different points in an electrical circuit. Voltmeters have a high internal resistance and are connected across (parallel to) the points where voltage is to be measured.

Watt-hour (Wh) — A unit of electrical energy equivalent to a power consumption of one watt for one hour.

Watt (W) — A unit of electrical power. One watt is equivalent to one joule per second, corresponding to the power in an electric circuit in which the potential difference is one volt and the current one ampere.

Wattmeter — The wattmeter is an instrument for measuring the electric power (or the supply rate of electrical energy) in watts of any given circuit.

Waveform — A graphical representation of electrical cycles which shows the amount of variation in amplitude over some period of time.

TYPES OF ELECTRICAL NETWORKS

The interconnection of various active and passive components in a prescribed manner to form a closed path is called an **electric circuit**. The system in which electric current can flow from the source to the load and then back to the other terminal of the source is referred to as an **electric circuit**. The main parts of an ideal **electric circuit** are:

1. Electrical sources for delivering electricity to the circuit and these are mainly electric generators and batteries
2. Controlling devices for controlling electricity and these are mainly switches, circuit breakers, MCBs, and potentiometer like devices etc.
3. Protection devices for protecting the circuit from abnormal conditions and these are mainly electric fuses, MCBs, switchgear systems.
4. Conducting path to carry electric current from one point to other in the circuit and these are mainly wires or conductors.
5. Load.

Thus, voltage and current are the two basic features of an **electric element**. Various techniques by which voltage and current across any element in any electric circuit are determined is called electric circuit analysis.

In this figure shows a simple electric circuit containing

- A battery of 30 V
- A carbon resistor of $5K\Omega$

Due to this, a current I , flows in the circuit and a potential drop of V volts appears across resistor.

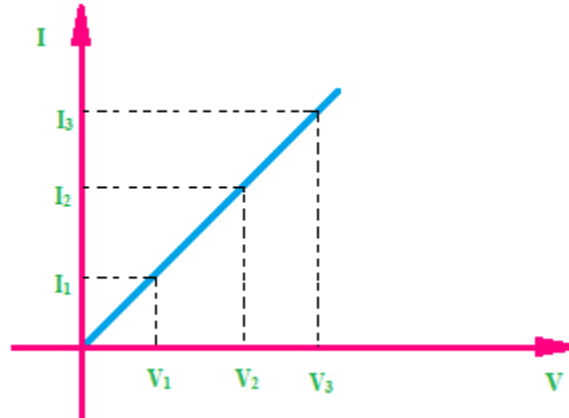
Basic Properties of Electric Circuits

- A circuit is always a closed path.
- A circuit always contains at least an energy source which acts as a source of electrons.
- The electric elements include uncontrolled and controlled source of energy, resistors, capacitors, inductors, etc.
- In an electric circuit flow of electrons takes place from negative terminal to positive terminal.
- Direction of flow of conventional current is from positive to negative terminal.
- Flow of current leads to potential drop across the various elements.

Types of Electrical Circuit:

1. Linear Circuit: When the flow of electrical current through an electrical circuit changes uniformly with the changes of voltage then that circuit is said to be as a Linear circuit. If the circuit consists of linear elements then the circuit will be Linear.

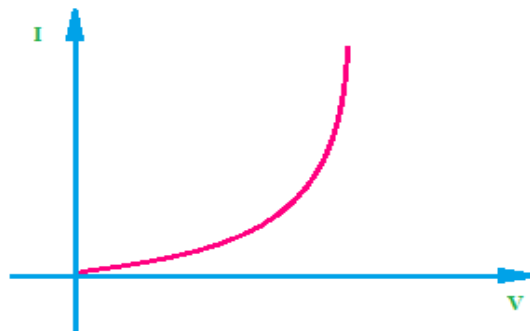
So we can also say if the parameters of an electrical circuit are constant, they do not change with the change of voltage and current then that circuit **is called linear circuit**. Actually, most of the cases the property of an electrical circuit depends on the property of connected elements.



The above diagram shows that the increase of flow of current is uniform to the increase of voltage.

2 Non-linear Circuit: When the flow of current through an electrical circuit changes ununiformly with the changes of voltage then the circuit is called **Non-linear Circuit**.

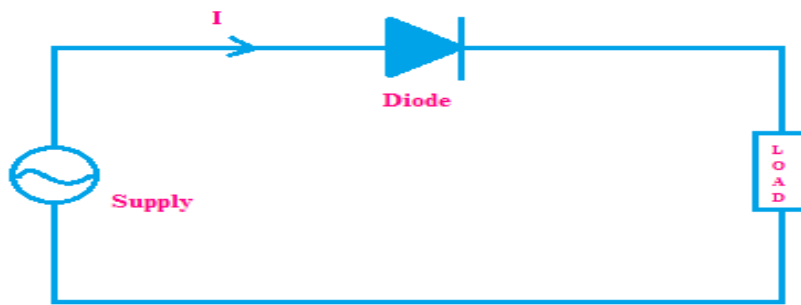
In the non-linear circuit the parameters are not constant they will change with the change of voltage and current.



3 Unilateral Circuit: When an electrical circuit changes its properties with the change of the direction of the flow of current then the circuit is called **Unilateral circuit**.

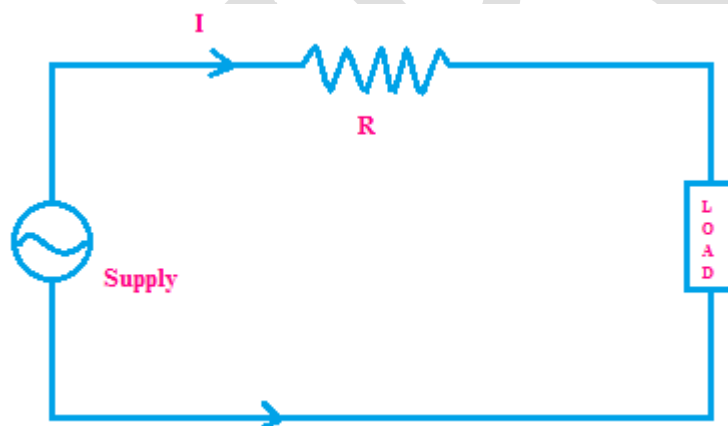
Suppose a resistive circuit whose main function is to oppose the flow of electrical current. If the opposition of the circuit to the flow of current is changed with the change of the direction of flow

of current then the circuit is said to be as Unilateral Circuit.



The circuit consisting of Diodes like the rectifier circuit is an **example of Unilateral Circuit**. In this circuit, the current can flow in one direction only because the diode allows the flow of current in one direction only.

4. Bilateral Circuit: If the properties of the electrical circuit do not change with the change of the direction of the flow of current then the circuit is called **Bilateral Circuit**.



The electrical Transmission Line is the **example of a bilateral circuit**.

5. Active Network:

A circuit which contains a source of energy is called **active network**. Voltage and current sources are energy sources.

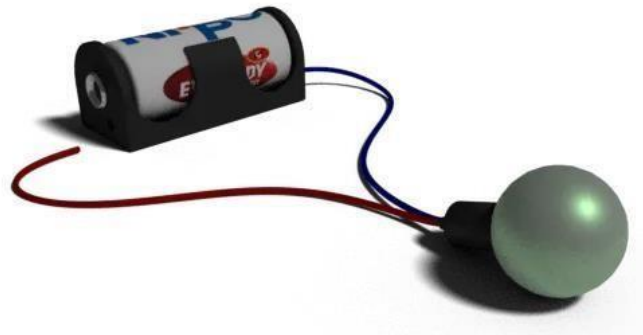
6. Passive Network:

A circuit which contains no energy source is called **passive network**. The passive network contains resistor, capacitor and inductor.

7. Open Circuit

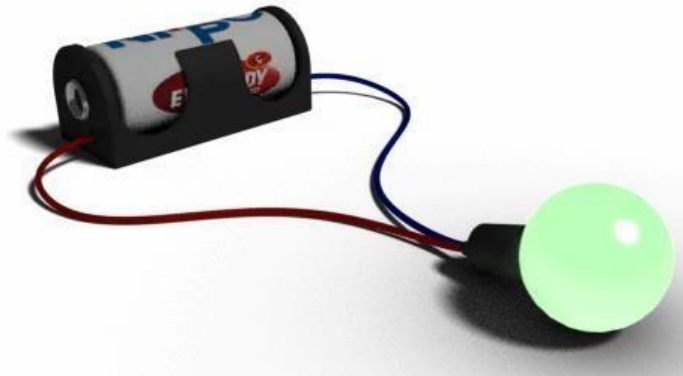
If due to disconnection of any part of an electric circuit if there is no flow of current through the

circuit, is said to be an **open circuited**.



& Closed Circuit

If there is no discontinuity in the circuit and current can flow from one part to another part of the circuit, the circuit is said to be **closed circuit**.



Short Circuit

If two or more phases, one or more phases and earth or neutral of AC system or positive and negative wires or positive or negative wires and earth of DC system touch together directly or connected together by a zero impedance path then the circuit is said to be **short circuited**.

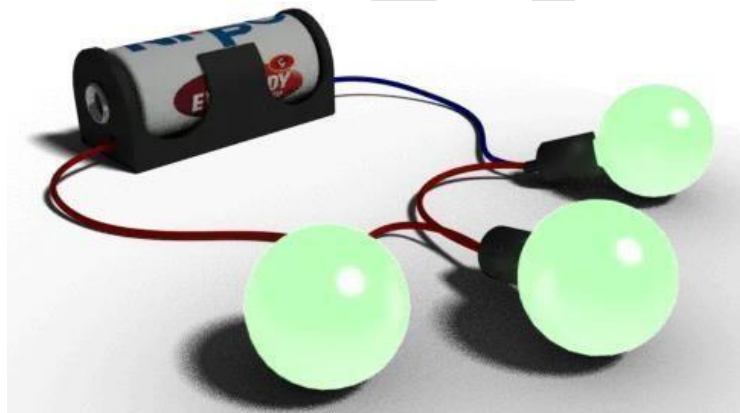


Electric circuits can be further categorized according to their structural features into either:

1. **Series Circuits**
2. **Parallel Circuits**
3. **Series Parallel Circuits**

Series Circuit

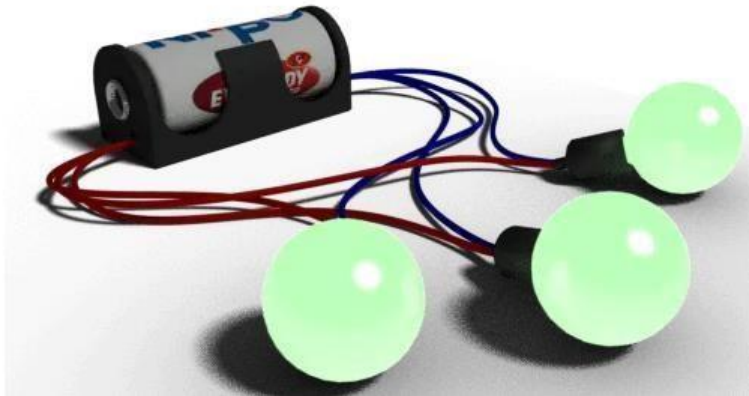
When all elements of a circuit are connected one after another in tail to head fashion and due to which there will be only one path of flowing current then the circuit is called **series circuit**. The circuit elements then are said to be series connected. In the series electrical circuit, same current flows through all element connected in series.



Parallel Circuit

If components are connected in such a way that the voltage drop across each component is same then it is known as **parallel circuit**. In **parallel circuit** the voltage drop across each component is same but the currents flowing through each component may differ. The total current is the sum of currents flowing through each element.

An example of a **parallel circuit** is the wiring system of a house. If one of the electric lamp burns out, current can still flow through the rest of the lights and appliances. In a parallel circuit the voltage is the same for all elements.



Types of Electronic Components

Electronic elements that make up a circuit are connected together by conductors to form a complete circuit. If these connecting conductors are ideal conductors (i.e. they have no resistance) then all parts of the circuit can be classified into two main categories depending on whether they deliver or absorb energy from the circuit:

- Active components
- Passive components

Electrical symbols are used to represent both active and passive components. An example of a basic circuit made up of two electronic elements has been illustrated below:

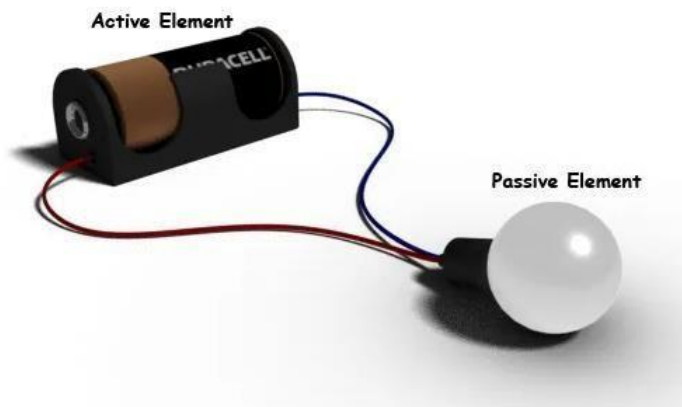


Active Components

An **active component** is an electronic component which supplies energy to a circuit. Active elements have the ability to electrically control electron flow (i.e. the flow of charge). All electronic circuits must contain at least one active component.

Common examples of active components include:

- Voltage sources
- Current sources (e.g. DC current source)
- Generators (such as alternators and DC generators)
- All different types of transistors (such as bipolar junction transistors, MOSFETS, FETs, and JFET)
- Diodes (such as Zener diodes, photodiodes, Schottky diodes, and LEDs)



Voltage Sources

A voltage source is an example of an active component in a circuit. When current leaves from the positive terminal of the voltage source, energy is being supplied to the circuit. As per the definition of an active element, a battery can also be considered as an active element, as it continuously delivers energy to the circuit during discharging.

Current Sources

A current source is also considered an active component. The current supplied to the circuit by an ideal current source is independent of circuit voltage. As a current source is controlling the flow of charge in a circuit, it is classified as an active element.

Transistors

Although not as obvious as a current or voltage source – transistors are also an active circuit component. This is because transistors are able to amplify the power of a signal (see our article on transistors as an amplifier if you want to know exactly how).

As this amplification is essentially controlling the flow of charge – transistors are hence

classified as an active component.

Passive Components

A **passive component** is an electronic component which can only receive energy, which it can either dissipate, absorb or store it in an electric field or a magnetic field. Passive elements do not need any form of electrical power to operate.

As the name ‘passive’ suggests – passive devices do not provide gain or amplification. Passive components cannot amplify, oscillate, or generate an electrical signal.

Common examples of passive components include:

- Resistors
- Inductors
- Capacitors
- Transformers

Resistors

A resistor is taken as a passive element since it can not deliver any energy to a circuit. Instead resistors can only receive energy which they can dissipate as heat as long as current flows through it.



Inductors

An inductor is also considered as passive element of circuit, because it can store energy in it as a magnetic field, and can deliver that energy to the circuit, but not in continuous basis. The energy absorbing and delivering capacity of an inductor is limited and transient in nature. That is why an inductor is taken as a passive **element of a circuit**.

Capacitors

A capacitor is considered as a passive element because it can store energy in it as electric field. The energy dealing capacity of a capacitor is limited and transient – it is not actually supplying

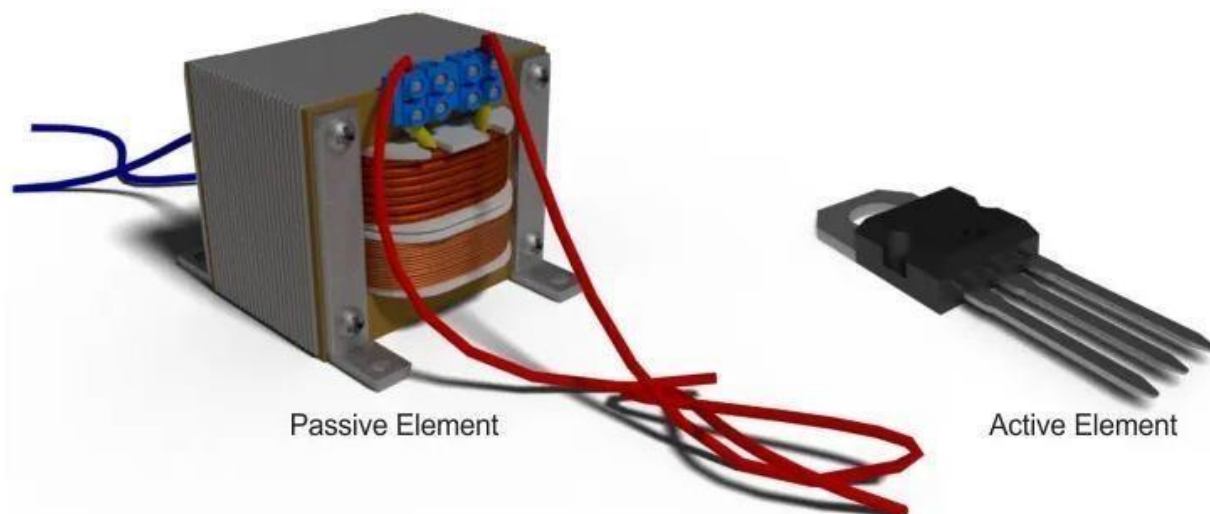
energy, it is storing it for later use.

As such it is not considered an active component since no energy is being supplied or amplified.

Transformers

A transformer is also a passive electronic component. Although this can seem surprising since transformers are often used to raise voltage levels – remember that power is kept constant.

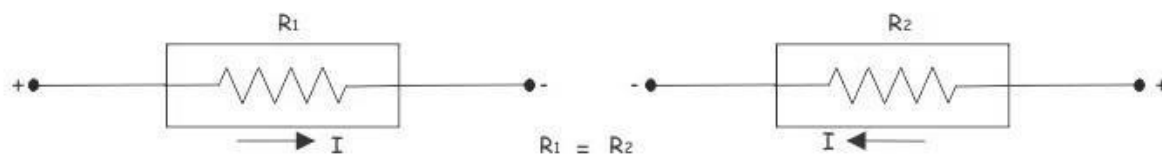
When transformers step up (or step down) voltage, power and energy remain the same on the primary and secondary side. As energy is not actually being amplified – a transformer is classified as a passive element.



Bilateral Elements

Conduction of current in both directions in a circuit element with same magnitude is termed as a **bilateral circuit element**. It offers some resistance to current flow in both directions.

Examples: Resistors, inductors, capacitors etc.

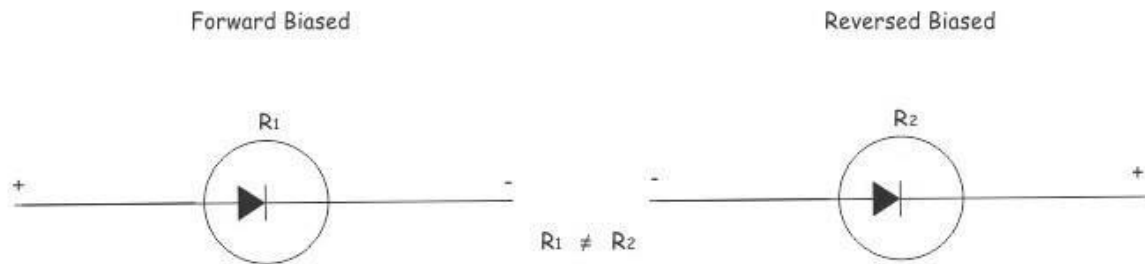


This figure shows that a bilateral circuit element can conduct from both sides and offers same resistance for current from either side.

Unilateral Elements

The **unilateral circuit element** does not offer same resistance to the current of either direction. The resistance of the **unilateral circuit element** is different for forward current than that of reverse current.

Examples: diode, transistor etc.



The figure above shows a diode as a unilateral circuit element. When diode is forward biased it offers very small resistance and conducts. While it is reverse biased, it offers very high resistance and doesn't conduct. The circuit element can be categorized in another manner, such as **lumped and distributed circuit elements**.

Lumped Elements

When the voltage across and current through the element don't vary with dimension of the element, it is called **lumped circuit elements**.

Examples: Resistor connected in any electrical circuit.

Distributed Elements

When the voltage across and current through the element change with dimensions of the element, it is called **distributed circuit element**.

Examples: Resistance of a transmission line. It varies with the length of the line.

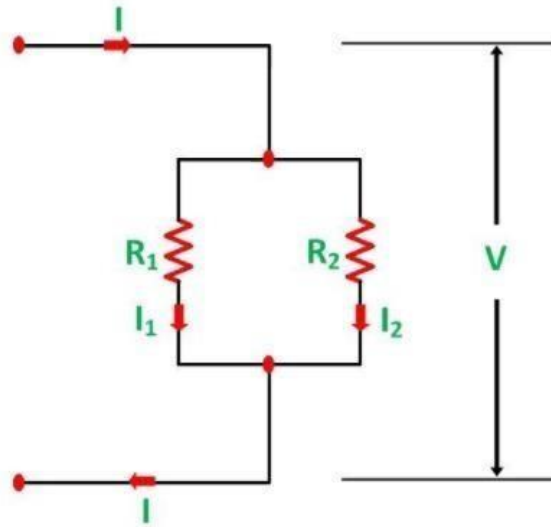
Comparison Chart of Active and Passive Elements

BASIS	ACTIVE COMPONENTS	PASSIVE COMPONENT
Nature of source	Active components deliver power or energy to the circuit.	Passive elements utilizes power or energy in the circuit.
Examples	Diodes, Transistors, SCR, Integrated circuits etc.	Resistor, Capacitor, Inductor etc.
Function of the component	Devices which produce energy in the form of voltage or current.	Devices which stores energy in the form of voltage or current.
Power Gain	They are capable of providing power gain.	They are incapable of providing power gain.
Flow of current	Active components can control the flow of current.	Passive components cannot control the flow of the current.
Requirement of external source	They require an external source for the operations.	They do not require any external source for the operations.
Nature of energy	Active components are energy donor.	Passive components are energy acceptor.

CURRENT DIVISION AND VOLTAGE DIVISION RULE

Current Division Rule

A parallel circuit acts as a current divider as the current divides in all the branches in a parallel circuit, and the voltage remains the same across them. The current division rule determines the current across the circuit impedance. The current division is explained with the help of the circuit shown below:



The current I has been divided into I_1 and I_2 into two parallel branches with the resistance R_1 and R_2 and V is the voltage drop across the resistance R_1 and R_2 .

As we know,

$$V = IR \dots \dots \dots (1)$$

Then the equation of the current is written as:

$$I_1 = \frac{V}{R_1} \quad \text{and} \quad I_2 = \frac{V}{R_2}$$

Let the total resistance of the circuit be R and is given by the equation shown below:

$$R = \frac{R_1 R_2}{R_1 + R_2} \dots \dots \dots (2)$$

Equation (1) can also be written as:

$$I = V/R \dots \dots \dots (3)$$

Now, putting the value of R from the equation (2) in the equation (3) we will get

$$I = \frac{V (R_1 + R_2)}{R_1 R_2} \dots \dots \dots (4)$$

But

$$V = I_1 R_1 = I_2 R_2 \dots\dots\dots(5)$$

Putting the value of $V = I_1 R_1$ from the equation (5) in equation (4), we finally get the equation as:

$$I = \frac{I_1 R_1 (R_1 + R_2)}{R_1 R_2} = \frac{I_1}{R_2} (R_1 + R_2) \dots\dots\dots(6)$$

And now considering $V = I_2 R_2$ the equation will be:

$$I = \frac{I_2 R_2 (R_1 + R_2)}{R_1 R_2} = \frac{I_2}{R_1} (R_1 + R_2) \dots\dots\dots(7)$$

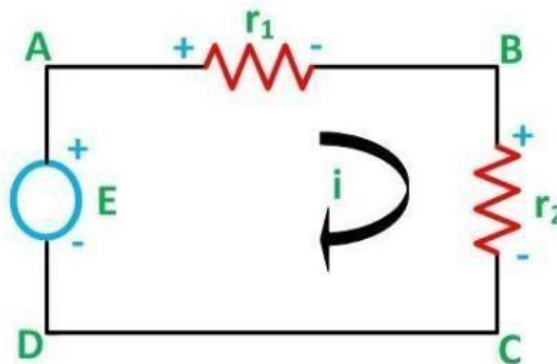
Thus, from the equation (6) and (7) the value of the current I_1 and I_2 respectively is given by the equation below:

$$I_1 = I \frac{R_2}{R_1 + R_2} \quad \text{and} \quad I_2 = I \frac{R_1}{R_1 + R_2}$$

Thus, in the current division rule, it is said that the current in any of the parallel branches is equal to the ratio of opposite branch resistance to the total resistance, multiplied by the total current.

Voltage Division Rule

The voltage division rule can be understood by considering a series circuit shown below. In a series circuit, voltage is divided, whereas the current remains the same.



Let us consider a voltage source E with the resistance r_1 and r_2 connected in series across it.

As we know,

$$I = V/R \text{ or we can say } I = E/R$$

Therefore, the current (i) in the loop ABCD will be:

$$i = \frac{E}{r_1 + r_2} \dots \dots \dots (8)$$

and $r_1 = ir_1$

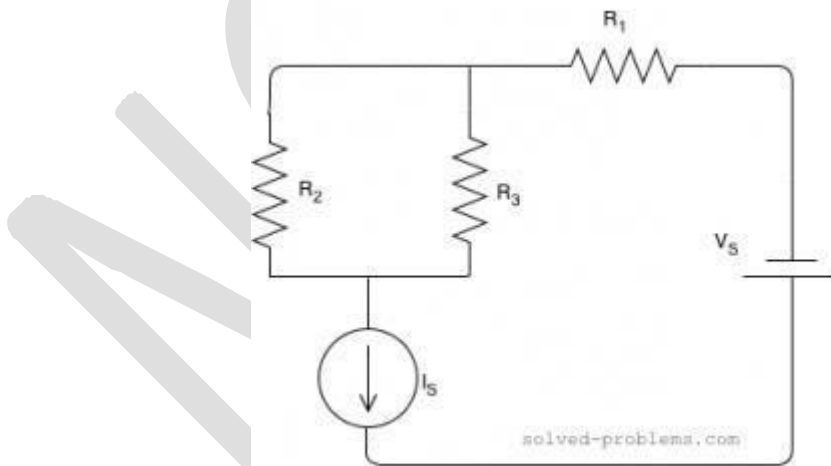
By putting the value of I from equation (8) in equation (9) the voltage across the resistance r_1 and r_2 respectively are given by the equation shown below as:

$$E_1 = \frac{Er_1}{r_1 + r_2} \text{ and } E_2 = \frac{Er_2}{r_1 + r_2}$$

Thus, the voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

Example Problems

Find current of resistors, use the current division rule.



Suppose that $R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 1\Omega$, $I_S = 5A$ and $V_S = 4V$

Solution:

R_2 and R_3 are parallel. The current of I_S is passing through them and it is actually divided between them. The branch with lower resistance has higher current because electrons can pass through that easier than the other branch. Using the current division rule, we get

$$I_{R_2} = \frac{R_3}{R_2 + R_3} \times I_S = \frac{1}{1+4} \times 5 = 1A$$
$$I_{R_3} = \frac{R_2}{R_2 + R_3} \times I_S = \frac{4}{1+4} \times 5 = 4A.$$

Note that $I_{R_2} < I_{R_3}$ because $R_2 > R_3$.

CONCEPT OF SOURCE TRANSFORMATION

Source Transformation simply means replacing one source by an equivalent source. A practical voltage source can be transformed into an equivalent practical current source and similarly a practical current source into voltage source.

Any practical voltage source or simply a voltage source consists of an ideal voltage source in series with an internal resistance or impedance (for an ideal source this impedance will be zero), the output voltage becomes independent of the load current. Cells, batteries and generators are the example of the voltage source.

For any practical current source or simply current source, there is an ideal current source in parallel with the internal resistance or impedance, for ideal current source this parallel impedance is infinity.

The semiconductor devices like transistors, etc. are treated as a current source or an output produce by the direct or alternating voltage source is called direct and alternating current source, respectively.

The voltage and current source are mutually transferable or in other words the source transformation i.e. voltage to the current source and current to a voltage source can be done. Let us understand this by considering a circuit given below:

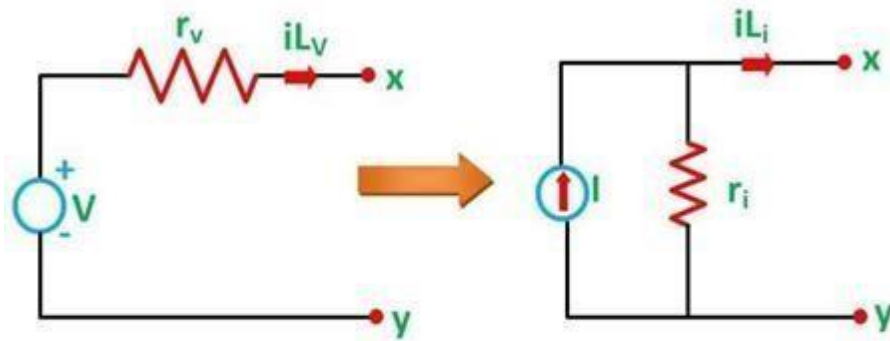


Figure A

Figure B

Figure A represents a practical voltage source in series with the internal resistance r_v , while figure B represents a practical current source with parallel internal resistance r_i

For the practical voltage source the load current will be given by the equation:

$$i_{L_v} = \frac{V}{r_v + r_L} \dots \dots \dots (1)$$

Where,

i_{L_v} is the load current for the practical voltage source

V is the voltage

r_v is the internal resistance of the voltage source

r_L is the load resistance

It is assumed that the load resistance r_L is connected at the terminal x-y. Similarly for the practical current source, the load current is given as:

$$i_{L_i} = I \frac{r_i}{r_i + r_L} \dots \dots \dots (2)$$

Where,

I_i is the load current for the practical current source

I is the current

r_i is the internal resistance of the current source

r_L is the load resistance connected across the terminal x-y in the figure B

Two sources become identical, when we will equate equation (1) and equation (2)

$$\frac{V}{r_v + r_L} = I \frac{r_i}{r_i + r_L}$$

However, for the current source, the terminal voltage at x-y would be $I r_i$, x-y terminal are open.
i.e.

$$V = I \times r_i$$

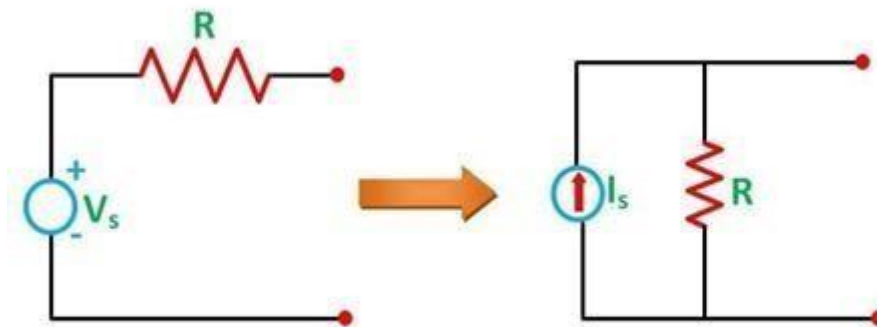
Therefore, we will get,

$$r_v + r_L = r_i + r_L \quad \text{or}$$

$$r_v = r_L$$

Therefore, for any practical voltage source, if the ideal voltage is V and internal resistance be r_v , the voltage source can be replaced by a current source I with the internal resistance in parallel with the current source.

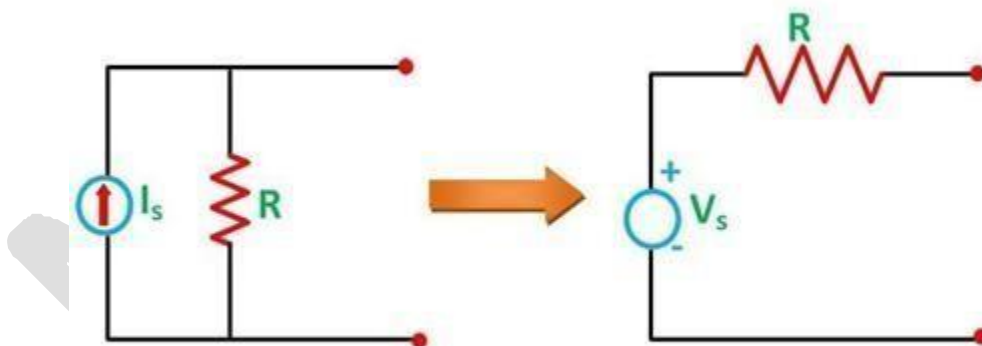
Source Transformation: Conversion of Voltage Source into Current Source



When the voltage source is connected with the resistance in series and it has to be converted into the current source than the resistance is connected in parallel with the current source as shown in the above figure.

Where $I_s = V_s / R$

Conversion of Current Source into Voltage Source



In the above circuit diagram a current source which is connected in parallel with the resistance is transformed into a voltage source by placing the resistance in series with the voltage source.

Where, $V_s = I_s / R$

Example 1:

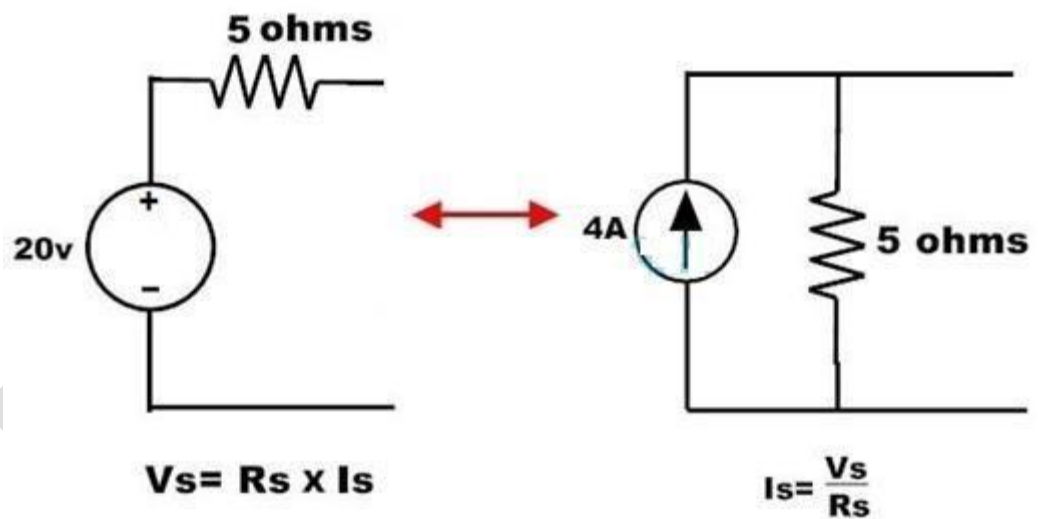
Consider the below voltage source circuit with a voltage of 20 V and a internal resistance of 5 ohms. This circuit is transformed into the current source by placing a resistor of the same value with a current source. This current source value can be determined by,

$$I_s = V_s/R_s$$

$$= 20/5$$

$$= 4 \text{ amps}$$

The equivalent current source with a current of 4A and parallel resistor of 5 ohms is shown below.



Example 2:

Consider the below example for current source transformation, where current source is of 10A with a parallel resistance of 3 ohms. To calculate the value of voltage in voltage source apply the simple ohms law, then,

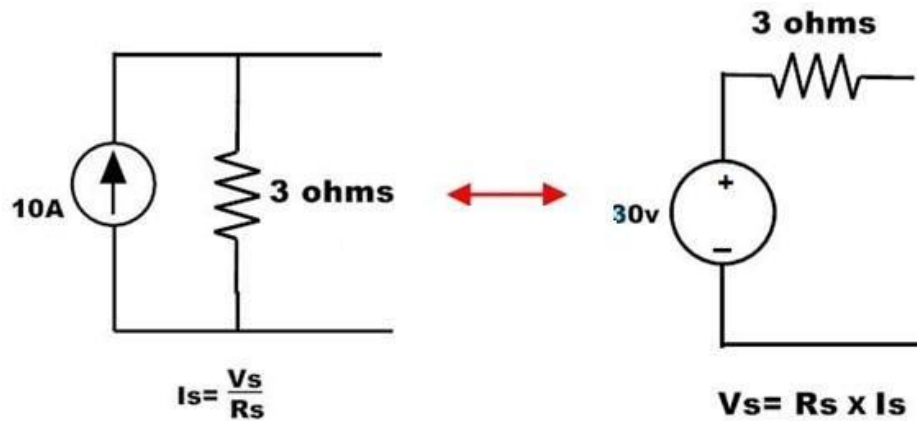
$$V_s = I_s * R_s$$

$$V_s = 10 * 3$$

$$= 30 \text{ Volts.}$$

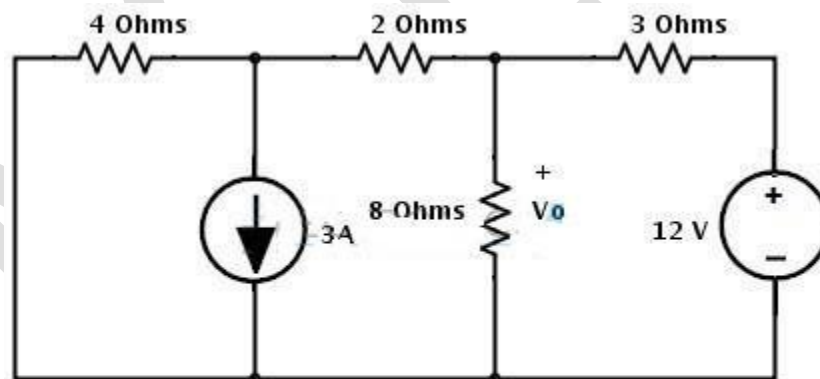
Therefore the equivalent voltage source of this transformation consists a voltage source 30 V

with a series resistance 3 ohms.

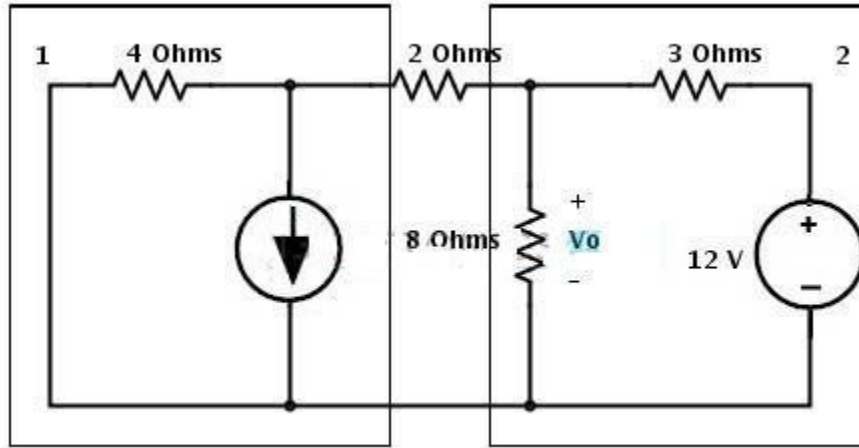


Example 3:

- Consider the below example where we have to find the voltage V_o by applying the source transformation. This circuit consists of both current and voltage sources. Let us see how we can apply source transformation to simplify the circuit below.



- In the circuit there are two areas where we can apply the source transformation since current source has a parallel resistor and voltage source has a series resistor as shown in figure. So these configurations are necessary requirements to apply the source transformation.



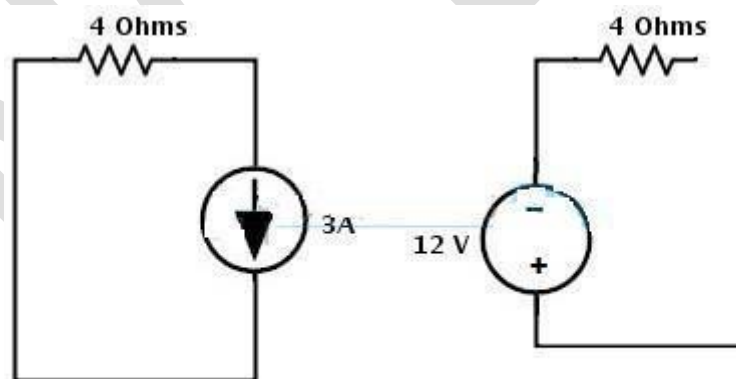
- First, consider the current source with a parallel resistance of 4 ohms. This current source can be transformed into a voltage source by 4 ohms series with a voltage source and voltage source value is determined as

$$V_s = I_s * R$$

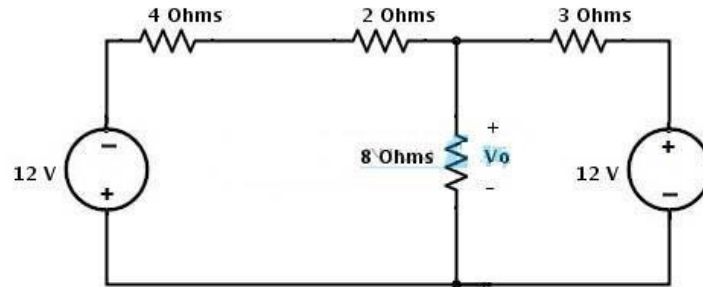
$$= 3 * 4$$

$$= 12 \text{ Volts}$$

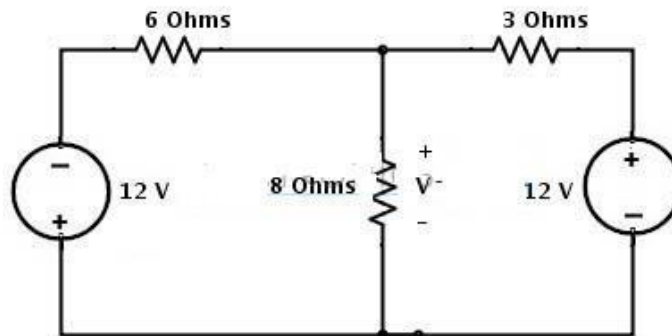
Consider the direction of the current as it downwards so the voltage terminals in voltage source are also changes as shown in figure.



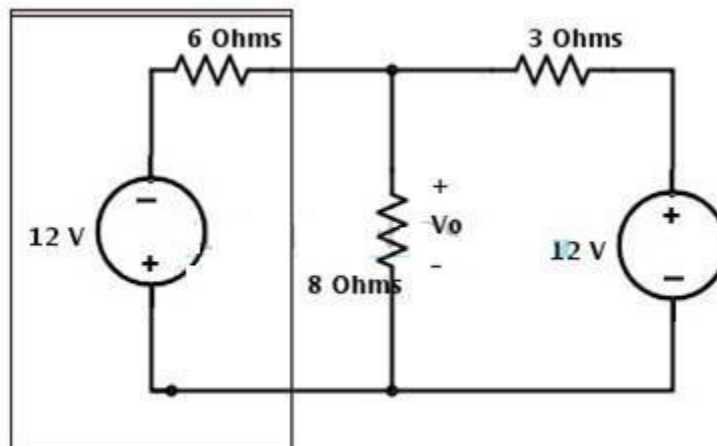
- Place the above voltage source with a series resistance in the circuit, then we get below figure.



- Resistors 4 ohms and 2 ohms are in series , hence the total series resistance will be 6 ohms as shown below.



- Again the voltage source of 12V with series resistor 6 ohms can be transformed into a current source. Therefore consider to transform it.



- This 12V voltage source with 6 ohm resistor combination can be converted into the current source by placing 6 ohms resistor in parallel with a current source. And the value

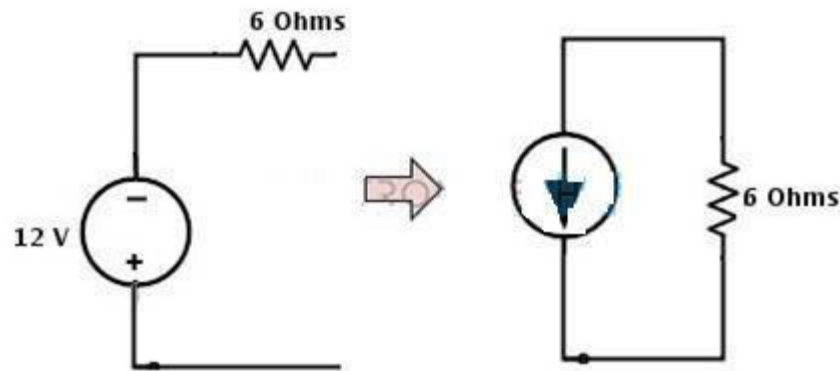
of current in current source can be determined as

$$I_s = V_s/R$$

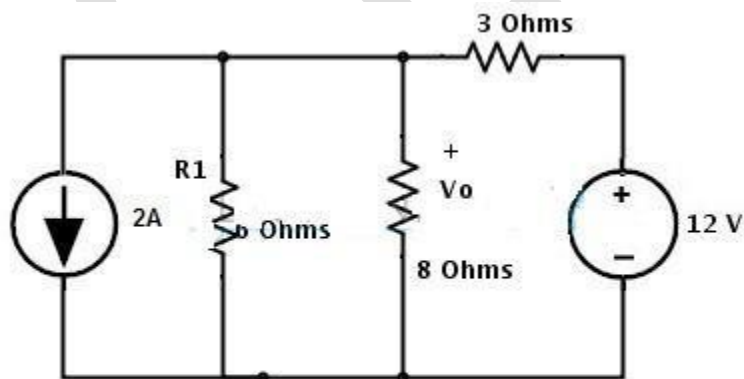
$$= 12/6$$

$$= 2\text{Amps}$$

The direction of current flow is represented in below figure.



- Insert the above current source in the main circuit, then we get



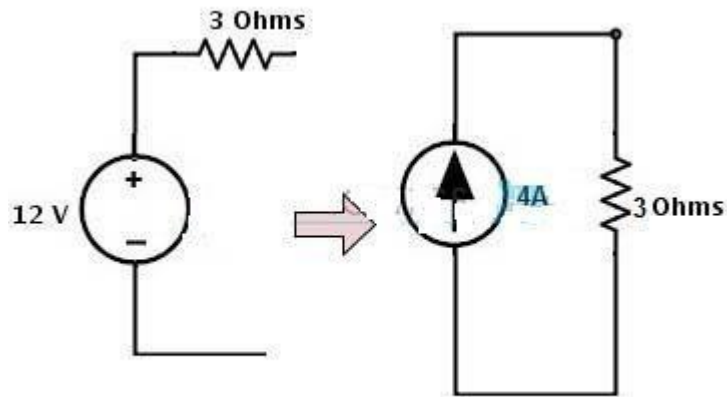
- On the right hand side, there is a voltage source with a 3 ohms resistor so this can be transformed into a current source by placing a 3 ohm resistor in parallel with a current source and this current source value is calculated as

$$I_s = V_s/R_s$$

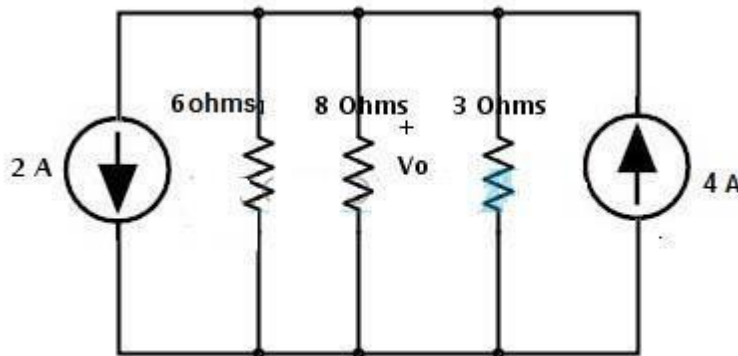
$$= 12/3$$

$$= 4 \text{ Amps}$$

The direction of current in current source is shown in figure.



- Insert the above current source in simplified circuit, then we get a final circuit as



From the above simplified circuit the current sources are appeared to be opposite to each other. The node current through the circuit will be

$$I_s = I_1 - I_2$$

$$= 4 - 2$$

$$= 2 \text{ amps}$$

By applying the divider rule, the current through the resistor 8 ohms is

$$I_o = I_s * (1/R_o / ((1/R_o) + (1/R_1) + (1/R_2)))$$

$$= 2 * (1/8 / ((1/8) + (1/6) + (1/3)))$$

$$= 0.4 \text{ Amps}$$

Therefore, the voltage across the resistor 8 ohms is

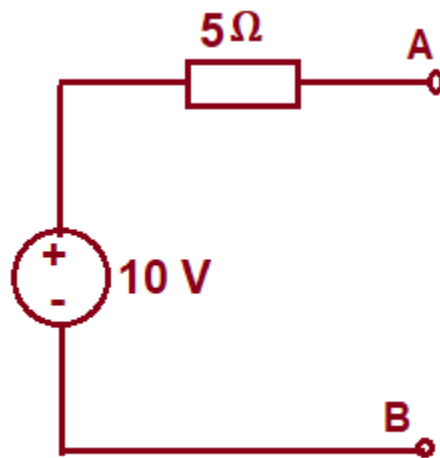
$$V_o = I_o * R_o$$

$$= 0.4 * 8$$

$$= 3.2 \text{ Volts}$$

Example 4:

Obtain an equivalent current source for the given voltage source:



Solution:

Let us first short terminals A & B and then find the current through the circuit. The current I through the circuit will be given as below.

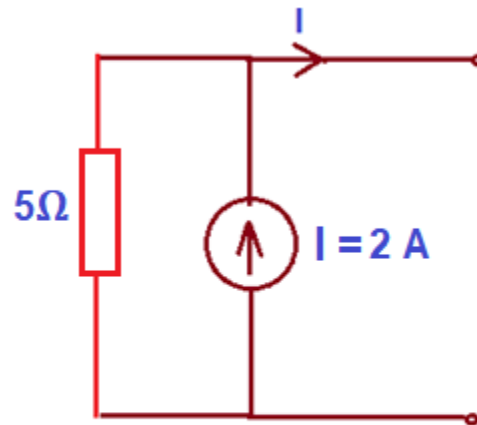
$$I = V/R$$

$$= 10/5$$

$$= 2 \text{ A}$$

This means, the equivalent current source should be capable of supplying 2 A of current. Hence strength of source will be 2 A. Again, the internal impedance / resistance of voltage source is 5 Ω , this resistance should be connected in parallel with the current source. Therefore, the

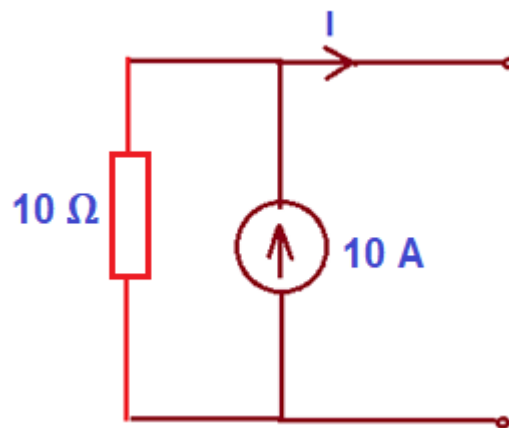
equivalent current source is given as below.



Equivalent Current Source

Example 5:

Convert the following current source to equivalent voltage source.



Solution:

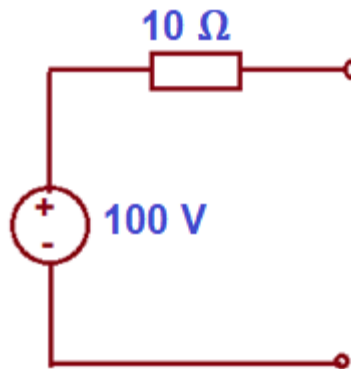
First of all, find the voltage across the terminals of the source while keeping the source terminal open. This voltage (V) is given as

$$V = IR$$

$$= 10 \times 10$$

$$= 100\text{ Volt}$$

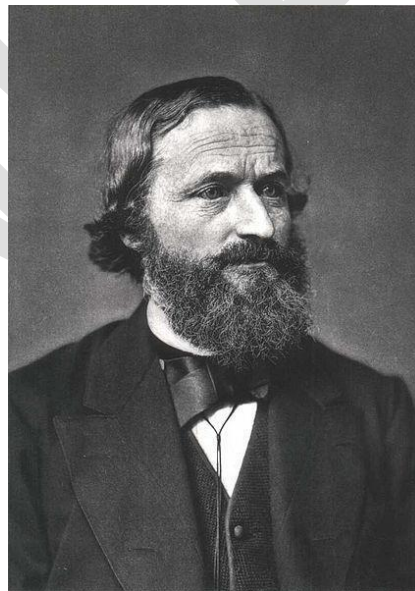
Thus, the strength of voltage source will be 100 V. The internal series resistance of this source will be equal to the resistance of current source i.e. 10 Ω . Therefore, equivalent voltage source is shown as below.



KIRCHHOFF'S LAW

Kirchhoff's circuit laws lie at the heart of circuit analysis. With the help of these laws and the equation for individual components (resistor, capacitor and inductor), we have the basic tool to start analyzing circuits. In this article, we will discuss Kirchhoff's current and voltage law and how to employ them in circuit analysis.

History about Gustav Robert Kirchhoff



Gustav Robert Kirchhoff(1824-1887)

Gustav Robert Kirchhoff, a German physicist, was born on March 12, 1824, in Königsberg, Prussia. His first research topic was on the conduction of electricity. This research led to Kirchhoff formulating the Laws of Closed Electric Circuits in 1845. These laws were eventually named after Kirchhoff and are now known as Kirchhoff's Voltage and Current Laws. Since these laws apply to all electric circuits, understanding their fundamentals is paramount in the understanding of how an electronic circuit functions. Although these laws have elevated Kirchhoff in the field of Electrical Engineering, he has additional discoveries. He was the first person to verify that an electrical impulse travelled at the speed of light. Furthermore, Kirchhoff made a major contribution to the study of spectroscopy and he advanced the research into blackbody radiation.

Many of the electrical circuits are complex in nature and the computations required to find the unknown quantities in such circuits, using simple Ohm's law and series/parallel combination simplifying methods is not possible. Therefore, in order to simplify these circuits Kirchhoff's laws are used.

These laws are the fundamental analytical tools that are used to find the solutions of voltages and currents in an electric circuit whether it can be AC or DC. Elements in an electric circuit are connected in numerous possible ways, thus to find the parameters in an electrical circuit these laws are very helpful.

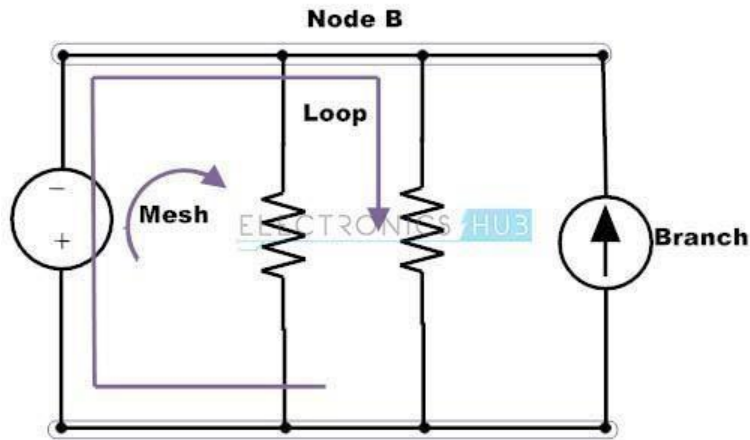
Before going to know more about Kirchhoff's law, we have to consider some of the terms related to electric circuits.

Node : Node or junction is a point in the circuit where two or more electrical elements are connected. This specifies a voltage level with a reference node in a circuit.

Branch: The continuous conducting path between two junctions which contains electrical element in a circuit is referred as branch.

Loop: In an electrical circuit a loop is an independent closed path in a circuit that follows the sequence of branches in such a way that it must start and ends with same node and it shouldn't touch any other junction or node more than once.

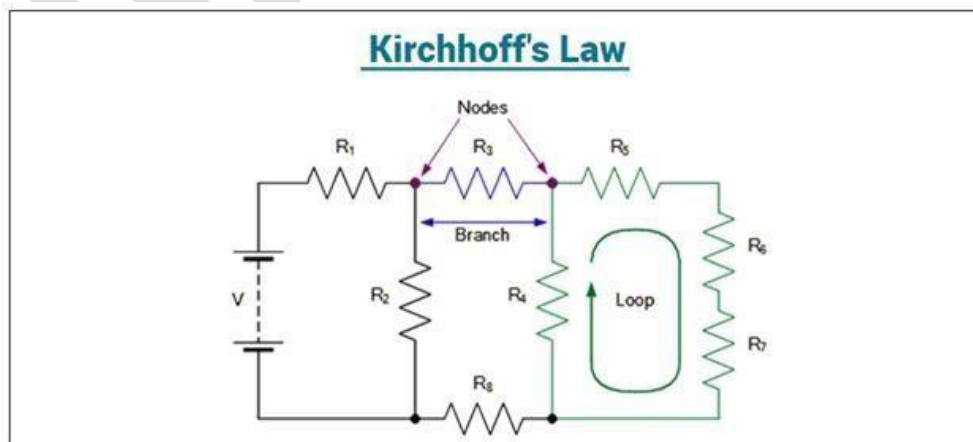
Mesh: In an electrical circuit mesh is a loop that doesn't contain any other loop in its interior.



What are Kirchhoff's Law?

In 1845, a German physicist, Gustav Kirchhoff developed a pair of laws that deal with the conservation of current and energy within electrical circuits. These two laws are commonly known as Kirchhoff's Voltage and Current Law. These laws help in calculating the electrical resistance of a complex network or impedance in case of AC and the current flow in different streams of the network. In the next section, let us look at what these laws state.

Kirchhoff's laws are a set of laws that quantify how current flows through a circuit and how voltage varies around a loop in a circuit. They are used to govern the conservation of charge and energy in standard electrical circuits. Two significant circuit laws are applied in every simple and complex electrical circuit in physics. These laws were postulated in 1845 by German physicist Gustav Kirchhoff. The proof of Kirchhoff's law can be obtained by using Maxwell's equations.



What do Kirchhoff's laws state?

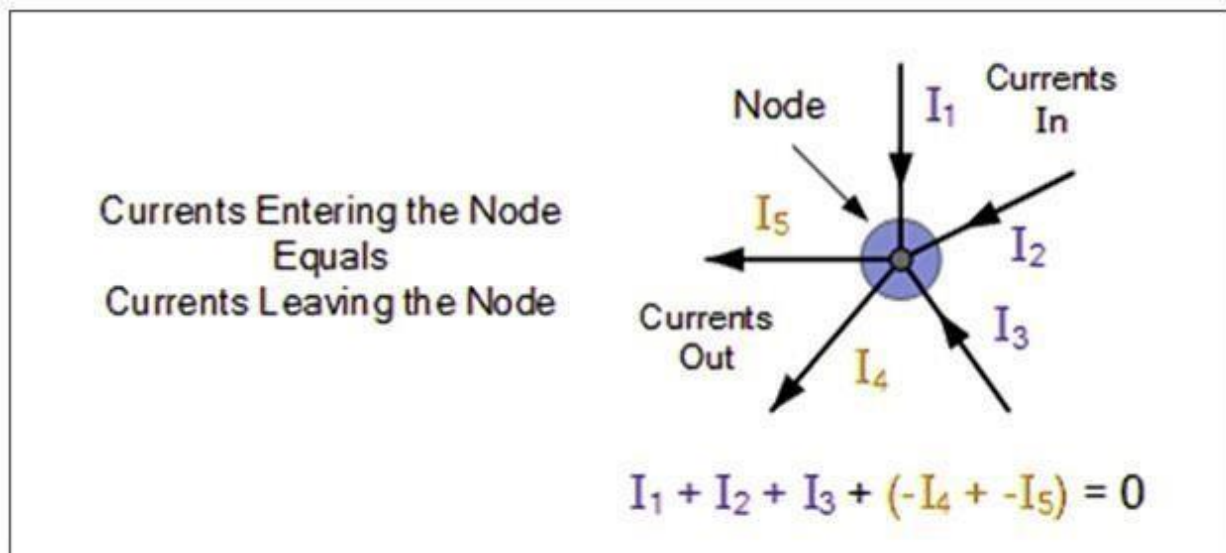
- Kirchhoff's Current Law goes by several names as Kirchhoff's First Law and Kirchhoff's Junction Rule. According to the Junction rule, in a circuit, the total of the currents in a junction is equal to the sum of currents outside the junction.
- Kirchhoff's Voltage Law goes by several names as Kirchhoff's Second Law and Kirchhoff's Loop Rule. According to the loop rule, the sum of the voltages around the closed loop is equal to null.

Kirchhoff's First Law

According to Kirchhoff's Current Law,

The total current entering a junction or a node is equal to the charge leaving the node as no charge is lost.

Put differently, the algebraic sum of every current entering and leaving the node has to be null. This property of Kirchhoff law is commonly called as Conservation of charge wherein, $I(\text{exit}) + I(\text{enter}) = 0$.



In the above figure, the currents I_1 , I_2 and I_3 entering the node is considered positive, likewise, the currents I_4 and I_5 exiting the nodes is considered negative in values. This can be expressed in the form of an equation:

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

The term Node refers to a junction or a connection of two or more current-carrying routes like cables and other components. Kirchhoff's current law can also be applied to analyze parallel

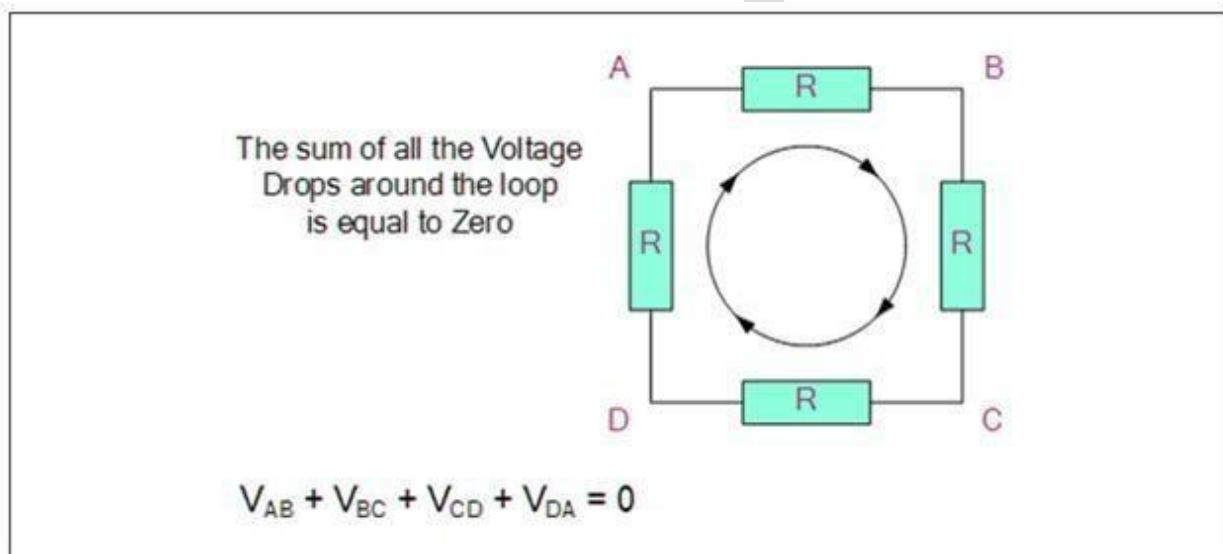
circuits.

Kirchhoff's Second Law

According to Kirchhoff's Voltage Law,

The voltage around a loop equals to the sum of every voltage drop in the same loop for any closed network and also equals to zero.

Put differently, the algebraic sum of every voltage in the loop has to be equal to zero and this property of Kirchhoff's law is called as conservation of energy.



When you begin at any point of the loop and continue in the same direction, note the voltage drops in all the direction either negative or positive and return to the same point. It is essential to maintain the direction either counterclockwise or clockwise; else the final voltage value will not be equal to zero. The voltage law can also be applied in analyzing circuits in series.

When either AC circuits or DC circuits are analysed based on Kirchhoff's circuit laws, you need to be clear with all the terminologies and definitions that describe the circuit components like paths, nodes, meshes, and loops.

Differences Between Kirchhoff's Current and Voltage Laws

Kirchhoff's Current Law vs. Kirchhoff's Voltage Law	
KCL	KVL
States that the sum of all the currents entering a particular node is equal to the sum of all currents leaving the node	States that the sum of all the voltages around a closed path (loop) is zero
Nodal analysis is preferred to obtain node potentials as the currents entering/leaving the node can be expressed in terms of node potential	Loop analysis is preferred to obtain loop currents as loop potential differences can be expressed in terms of loop currents

Kirchhoff's Law Circuit Diagram

A circuit diagram consists of a source of current and voltage along with resistances and impedances, which can be in series, or parallel, or combination of the two. The polarity of the source is indicated by positive and negative signs, which automatically applies to the resistances.

Resistances in Series

Resistances are said to be in series when they are connected in a single path. The current from a source flows through all the resistances in a closed loop.

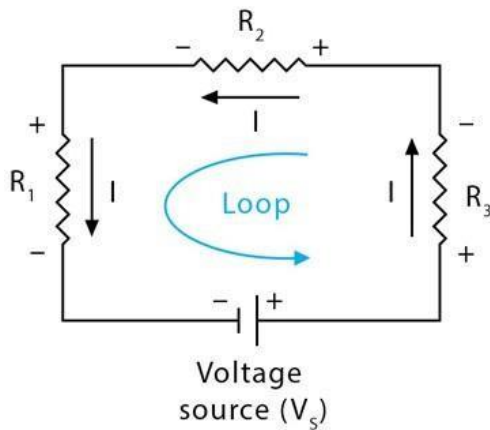
Resistances in Parallel

Resistances are said to be in parallel when the path branches and each branch consists of one resistance. The current from the source splits into different paths. The equation for replacing resistances in parallel is a bit more complicated.

Sign Convention

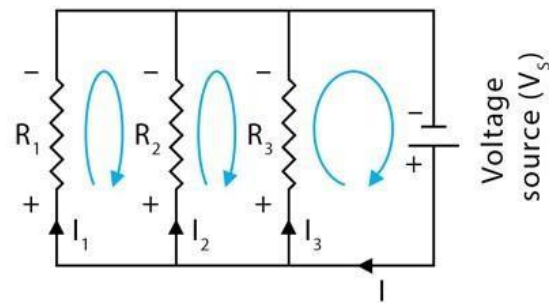
The sign convention for applying signs to the voltage polarities in KVL equations is as follows. When traversing the loop, if the positive terminal of a voltage difference is encountered before the negative terminal, the voltage difference will be interpreted as positive. If the negative terminal is encountered first, the voltage difference will be interpreted as negative.

Resistances in series



$$IR_1 + IR_2 + IR_3 - V_s = 0$$

Resistances in parallel



$$I = I_1 + I_2 + I_3$$

$$I_1 R_1 = I_2 R_2 = I_3 R_3 = V_s$$

Applications of Kirchhoff's Law

Kirchhoff's laws are applicable to analyze any circuit regardless of the composition and structure of it. Some of its applications include

- To find the unknown resistances, impedances, voltages, and currents (direction as well as value).
- In a branched circuit, currents passing each branch are determined by applying KCL at every junction and KVL in every loop.
- In a looped circuit, the current passing each independent loop is determined by applying KVL for each loop and calculating the currents in any resistance of the circuit.

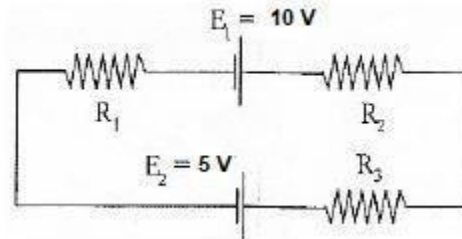
Limitations of Kirchhoff's Law

Kirchhoff's laws are limited in their applicability. They are valid for all cases in which total electric charge is constant in the region into consideration. Essentially, this is always true, so long as the law is applied for a specific point. Over a region, however, charge density may not be constant. Because the charge is conserved, the only way this is possible is if there is a flow of charge across the boundary of the region. This flow would result in current, thus violating Kirchhoff's laws.

Another limitation is that it works under the assumption that there is no fluctuating magnetic field in the closed-loop. Electric fields and emf could be induced, which causes Kirchhoff's laws to break in the presence of a variable magnetic field.

Kirchhoff's Law Solved Example

If $R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 6\Omega$, determine the electric current that flows in the circuit below.



Solution:

Following are the things that you should keep in mind while approaching the problem:

Following are the things that you should keep in mind while approaching the problem:

You need to choose the direction of the current. In this problem, let us choose the clockwise direction.

When the current flows across the resistor, there is a potential decrease. Hence, $V = IR$ is signed negative.

If the current moves from low to high then the source of emf I signed positive because of the charging of energy at the emf source. Likewise, if the current moves from high to low voltage (+ to -) then the source of emf I signed negative because of the emptying of energy at the emf source.

In this solution, the direction of the current is the same as the direction of clockwise rotation.

$$-IR_1 + E_1 - IR_2 - IR_3 - E_2 = 0$$

Substituting the values in the equation, we get

$$-2I + 10 - 4I - 6I - 5 = 0$$

$$-12I + 5 = 0$$

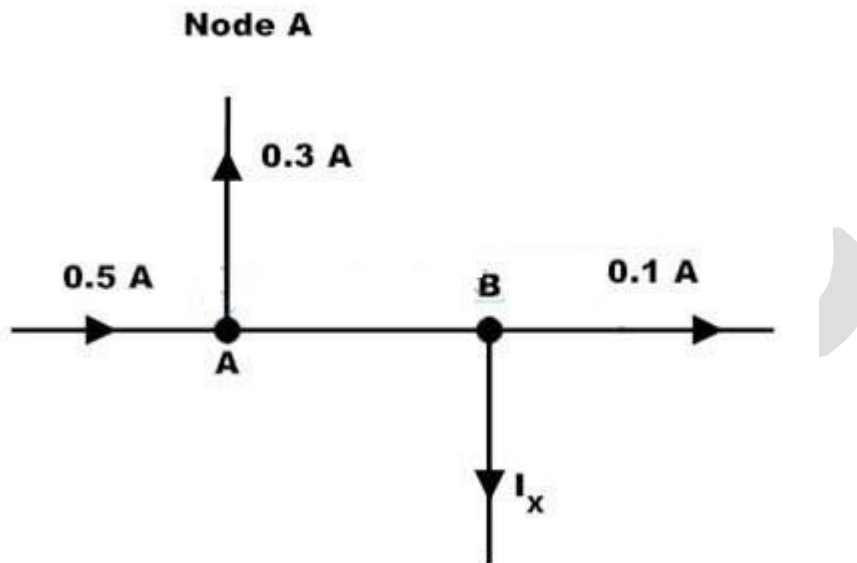
$$I = -5/-12$$

$$I = 0.416 \text{ A}$$

The electric current that flows in the circuit is 0.416 A. The electric current is signed positive which means that the direction of the electric current is the same as the direction of clockwise rotation. If the electric current is negative then the direction of the current would be in anti-clockwise direction.

Example Problem of KCL

Consider the below figure where we have to determine the currents I_{AB} and I_x by using KCL .



By applying Kirchhoff's Current Law at point A, we get

$$I_{AB} = 0.5 - 0.3$$

$$I_{AB} = 0.2 \text{ Amps}$$

Similarly by applying KCL at point B, we get

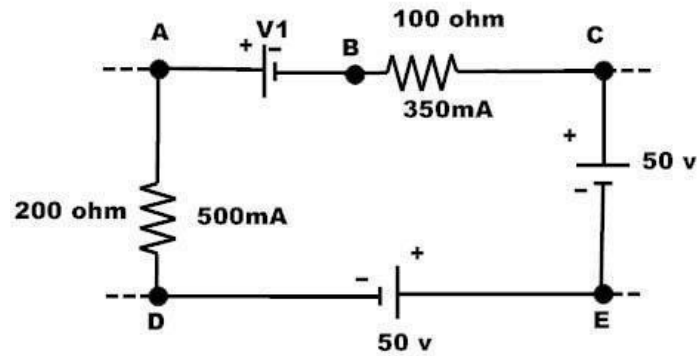
$$I_{AB} = 0.1 + I_x$$

$$0.2 = 0.1 + I_x$$

$$I_x = 0.2 - 0.1 = 0.1 \text{ Amps}$$

Kirchhoff's Voltage Law Example

1. Let us consider the single loop circuit which is shown below and assume the current flow direction as DEABCD closed path. In this circuit, by using KVL we have to find the voltage V_1 .



By applying KVL to this closed loop, we can write as

$$V_{ED} + V_{AE} + V_{BA} + V_{CB} + V_{DC} = 0$$

Where

Voltage of point E with respect to point D, $V_{ED} = -50 \text{ V}$

Voltage of point D with respect to point C, $V_{DC} = -50 \text{ V}$

Voltage of point A with respect to point E. $V_{AE} = I * R$

$$V_{AE} = 500\text{m} * 200$$

$$V_{AE} = 100 \text{ V}$$

Similarly Voltage at point C with respect to point B, $V_{CB} = 350\text{m} * 100$

$$V_{CB} = 35\text{V}$$

Consider voltage at point A with respect to point B, $V_{AB} = V_1$

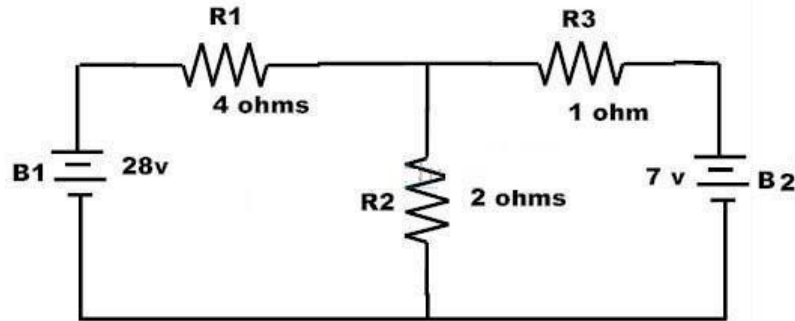
$$V_{BA} = -V_1$$

Then by using KVL

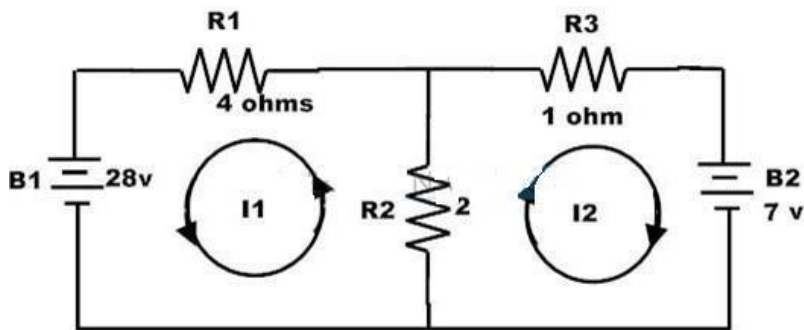
$$-50 + 100 - V_1 + 35 - 50 = 0$$

$$V_1 = 35 \text{ Volts}$$

2. Consider the below typical two loop circuit where we have to find the currents I_1 and I_2 by applying the Kirchhoff's laws.



There are two loops inside the circuit and consider the loop paths as shown in figure.



By applying KVL to these loops we get

For first loop,

$$2(I_1 + I_2) + 4I_1 - 28 = 0$$

$$6I_1 + 2I_2 = 28 \quad \text{--- (1)}$$

For second loop,

$$-2(I_1 + I_2) - 1I_2 + 7 = 0$$

$$-2I_1 - 3I_2 = -7 \quad \text{--- (2)}$$

By solving the above 1 and 2 equations we get,

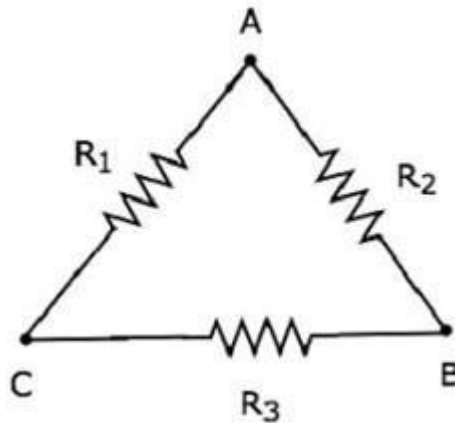
$$I_1 = 5A \text{ and } I_2 = -1A$$

STAR-DELTA TRANSFORMATIONS

For example, the resistors connected in either delta (δ) form or star form. In such situations, we have to **convert** the network of one form to the other in order to simplify it further by using series combination or parallel combination. In this chapter, let us discuss about the **Delta to Star Conversion**.

DELTA NETWORK

Consider the following **delta network** as shown in the following figure.



The following equations represent the **equivalent resistance** between two terminals of delta network, when the third terminal is kept open.

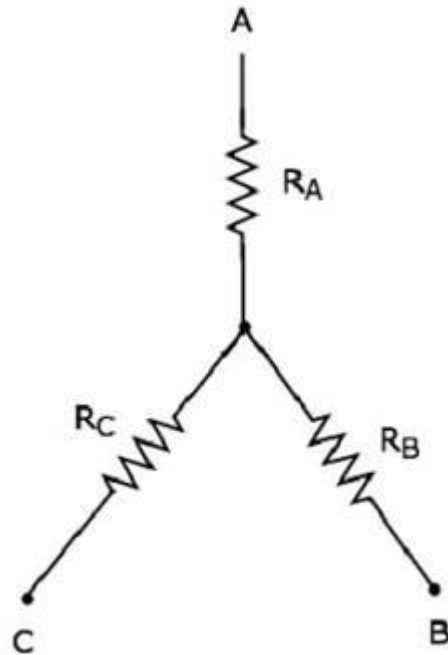
$$R_{AB} = \frac{(R_1 + R_3)R_2}{R_1 + R_2 + R_3}$$

$$R_{BC} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

$$R_{CA} = \frac{(R_2 + R_3)R_1}{R_1 + R_2 + R_3}$$

STAR NETWORK

The following figure shows the **equivalent star network** corresponding to the above delta network.



The following equations represent the **equivalent resistance** between two terminals of star network, when the third terminal is kept open.

$$R_{AB} = R_A + R_B$$

$$R_{BC} = R_B + R_C$$

$$R_{CA} = R_C + R_A$$

STAR NETWORK RESISTANCES IN TERMS OF DELTA NETWORK RESISTANCES

We will get the following equations by equating the right-hand side terms of the above equations for which the left-hand side terms are same.

$$R_A + R_B = \frac{(R_1 + R_3)R_2}{R_1 + R_2 + R_3} \quad \text{Equation 1}$$

$$R_B + R_C = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} \quad \text{Equation 2}$$

$$R_C + R_A = \frac{(R_2 + R_3)R_1}{R_1 + R_2 + R_3} \quad \text{Equation 3}$$

By adding the above three equations, we will get

$$2(R_A + R_B + R_C) = \frac{2(R_1R_2 + R_2R_3 + R_3R_1)}{R_1 + R_2 + R_3}$$

$$\Rightarrow R_A + R_B + R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1 + R_2 + R_3} \quad \text{Equation 4}$$

Subtract Equation 2 from Equation 4.

$$R_A + R_B + R_C - (R_B + R_C) = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1 + R_2 + R_3} - \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

$$R_A = \frac{R_1R_2}{R_1 + R_2 + R_3}$$

By subtracting Equation 3 from Equation 4, we will get

$$R_B = \frac{R_2R_3}{R_1 + R_2 + R_3}$$

By subtracting Equation 1 from Equation 4, we will get

$$R_C = \frac{R_3R_1}{R_1 + R_2 + R_3}$$

By using the above relations, we can find the resistances of star network from the resistances of delta network. In this way, we can convert a **delta network** into a **star network**.

DELTA NETWORK RESISTANCES IN TERMS OF STAR NETWORK RESISTANCES

Let us manipulate the above equations in order to get the resistances of delta network in terms of resistances of star network.

- **Multiply** each set of two equations and then **add**.

$$R_AR_B + R_BR_C + R_CR_A = \frac{R_1R_2^2R_3 + R_2R_3^2R_1 + R_3R_1^2R_2}{(R_1 + R_2 + R_3)^2}$$

$$\Rightarrow R_AR_B + R_BR_C + R_CR_A = \frac{R_1R_2R_3(R_1 + R_2 + R_3)}{(R_1 + R_2 + R_3)^2}$$

$$\Rightarrow R_AR_B + R_BR_C + R_CR_A = \frac{R_1R_2R_3}{R_1 + R_2 + R_3} \quad \text{Equation 4}$$

- By dividing Equation 4 with Equation 2, we will get

$$\frac{R_A R_B + R_B R_C + R_C R_A}{R_B} = R_1$$

$$\Rightarrow R_1 = R_C + R_A + \frac{R_C R_A}{R_B}$$

- By dividing Equation 4 with Equation 3, we will get

$$R_2 = R_A + R_B + \frac{R_A R_B}{R_C}$$

- By dividing Equation 4 with Equation 1, we will get

$$R_3 = R_B + R_C + \frac{R_B R_C}{R_A}$$

By using the above relations, we can find the resistances of delta network from the resistances of star network. In this way, we can convert **star network into delta network**.

Solved Examples on Star/Delta Transformation

Q1). Determine the resistance between the terminals A&B and hence find the current through the voltage source. Refer figure

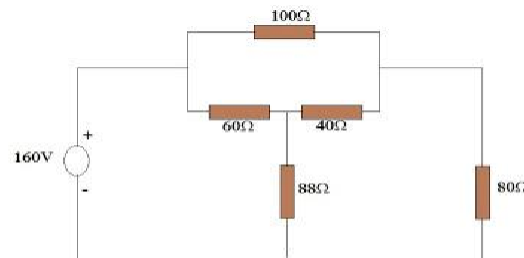


Figure 16.1

Answer:

See figure

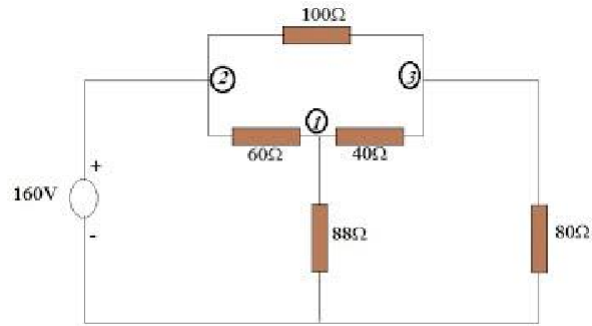


Figure 16.1 (a)

The resistors in between point 1, 2&3 are about to replace by a star connected system. Otherwise is difficult to find the total resistance.

So we have to use the delta to star transformation equations.

$$R_1 = R_{12}R_{31} / (R_{12}+R_{23}+R_{31})$$

$$R_1 = (60 \cdot 40) / (60+40+100)$$

$$R_1 = 12\Omega$$

$$R_2 = R_{23}R_{12} / (R_{12}+R_{23}+R_{31})$$

$$R_2 = (100 \cdot 60) / 200$$

$$R_2 = 30\Omega$$

$$R_3 = R_{31}R_{23} / (R_{12}+R_{23}+R_{31})$$

$$R_3 = (100 \cdot 40) / 200$$

$$R_3 = 20\Omega$$

So we can redraw the network as shown in figure

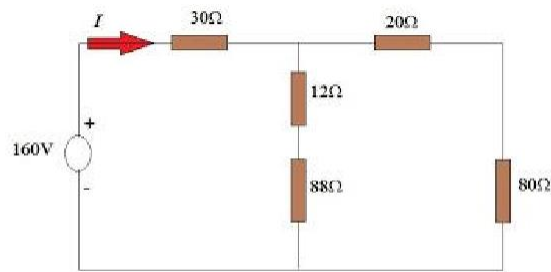


Figure 16.2

Now we can easily find the total resistance between A&B terminals

$$R_{\text{total}} = [(80+20)/(88+12)] + 30$$

$$R_{\text{total}} = 50 + 30$$

$$R_{\text{total}} = 80\Omega$$

Applying ohm's law to the total resistance,

$$I = V/R$$

$$I = 160\text{v}/80\Omega$$

$$I = 2\text{A}$$

Q2) Find the total resistance between A&B terminals for the network shown in figure 16.3

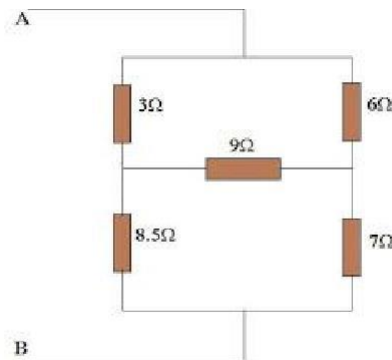


Figure 16.3

Answer:

See figure

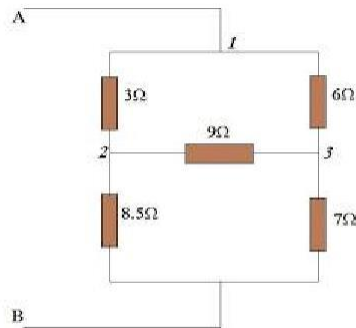


Figure 16.3(a)

We are about to replace the delta system by star system in between point 1, 2 & 3

So we have to use the delta to star transformation equations.

$$R_1 = R_{12}R_{31} / (R_{12}+R_{23}+R_{31})$$

$$R_1 = (3*6) / (3+6+9)$$

$$R_1 = 1\Omega$$

$$R_2 = R_{23}R_{12} / (R_{12}+R_{23}+R_{31})$$

$$R_2 = (9*3) / 18$$

$$R_2 = 1.5\Omega$$

$$R_3 = R_{31}R_{23} / (R_{12}+R_{23}+R_{31})$$

$$R_3 = (6*9) / 18$$

$$R_3 = 3\Omega$$

So now we can replace the system as shown in figure

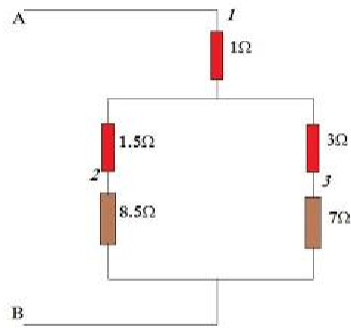


Figure 16.4

Now we can easily find the total resistance between A&B terminals

$$R_{AB} = (7\Omega + 3\Omega) + (8.5\Omega + 1.5\Omega) + 1\Omega$$

$$R_{AB} = 6\Omega$$

Q3). Find the total resistance between A&B terminals (R_{AB}) shown in figure

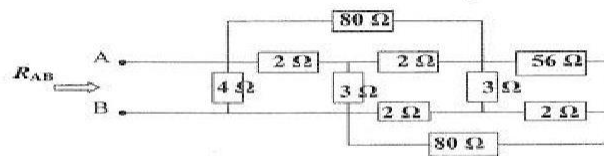


Figure 16.5

Answer:

You must understand that you have to use star/delta transformation for this problem. Unlike other problems, in this case it is not pointed out which system of resistance you must replace. So you yourself have to point it out.

This is very important. Though the tutorial problems guide you to find the replaceable systems, in practical level you will have to guide yourself manually. This means you must know how to choose the correct system to apply delta/star transformation.

See figure

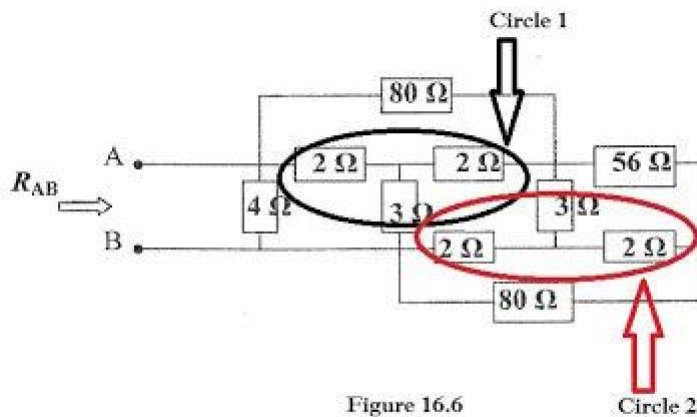


Figure 16.6

See the circled systems in the figure. You have to replace these systems with delta systems. If you see it carefully, you'll see that both systems are same (one is upside down of the other). So you don't need to find two different sets of delta systems. See figure 16.7

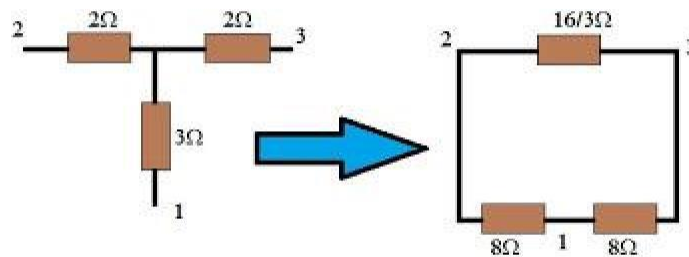


Figure 16.7

This figure shows you the star to delta transformation. As the required equation for transformation are given in my previous post, I've directly put the values for the delta system shown in the above figure. Steps for this calculation are shown below.

$$R_{12} = R_1 + R_2 + (R_1 R_2 / R_3)$$

$$R_{12} = 3 + 2 + (3 \cdot 2) / 2$$

$$R_{12} = 8\Omega$$

$$R_{23} = R_2 + R_3 + (R_2 R_3 / R_1)$$

$$R_{23} = 2 + 2 + (2 \cdot 2) / 3$$

$$R_{23} = 16/3 \Omega$$

$$R_{31} = R_3 + R_1 + (R_3 R_1 / R_2)$$

$$R_{13} = 3 + 2 + (3 \cdot 2) / 2$$

$$R_{13} = 8 \Omega$$

So we can redraw the network as shown in figure 16.8

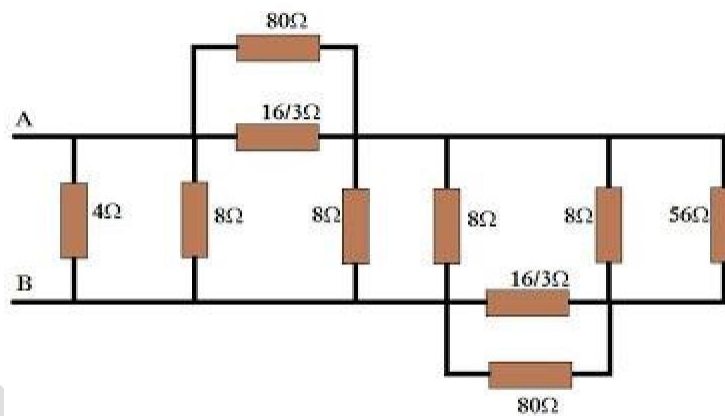


Figure 16.8

Now we can easily find the total resistance between A&B terminals. For your better understanding I've simplified the network. See figure

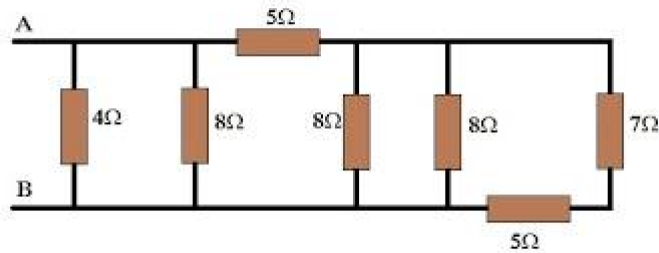


Figure 16.9

So now it is simple.

$$R_{AB} = \{ [(7+5) // 8 // 8] + 5 \} // 8 // 4$$

$$R_{AB} = (3 + 5) // 8 // 4$$

$$R_{AB} = 4 // 4$$

$$R_{AB} = 2\Omega$$

MESH ANALYSIS/MESH CURRENT METHOD

In circuit analysis, simple circuits can be analyzed by using the basic analyzing tools like ohms law, KVL and KCL. But for a complex circuit that consists of various controlled sources, these tools in addition with series and parallel methods are unreliable. Therefore, to find the variables of a branch in such circuit, nodal and mesh (or loop) analysis methods are used. By using these classical methods, circuit variables like voltage and currents are easily determined in any branch without a great difficulty. Let us see in detail about mesh analysis.

Mesh Analysis

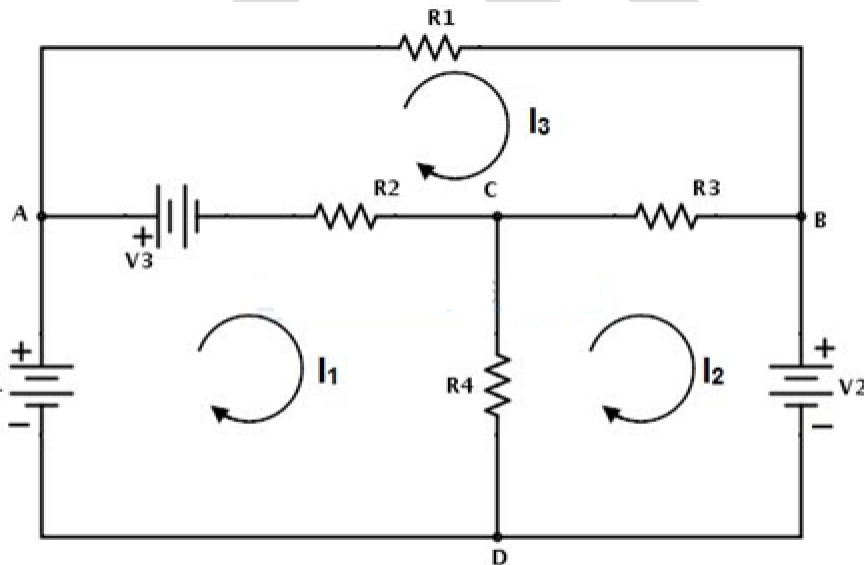
Mesh is a loop that doesn't consist of any other loop inside it. Mesh analysis technique, uses mesh currents as variables, instead of currents in the elements to analyse the circuit. Therefore, this method absolutely reduces the number of equations to be solved. Mesh analysis applies the Kirchhoff's Voltage Law (KVL) to determine the unknown currents in a given circuit. Mesh analysis is also called as mesh-current method or loop analysis. After finding the mesh currents using KVL, voltages anywhere in a given circuit can be determined by using

Ohms law.

Steps to Analyse the mesh analysis technique

- 1) Check whether there is a possibility to transform all current sources in the given circuit to voltage sources.
- 2) Assign the current directions to each mesh in a given circuit and follow the same direction for each mesh.
- 3) Apply KVL to each mesh and simplify the KVL equations.
- 4) Solve the simultaneous equations of various meshes to get the mesh currents and these equations are exactly equal to the number of meshes present in the network.

Consider the below DC circuit to apply the mesh current analysis, such that currents in different meshes can be found. In the below figure there are three meshes present as ACDA, CBDC and ABCA but the path ABDA is not a mesh. As a first step, the current through each mesh is assigned with the same direction as shown in figure.



Secondly, for each mesh we have to apply KVL. By applying KVL around the first loop or mesh we get

$$V_1 - V_3 - R_2 (I_1 - I_3) - R_4 (I_1 - I_2) = 0$$

$$V_1 - V_3 = I_1 (R_2 + R_4) - I_2 R_4 - I_3 R_2 \dots\dots\dots (1)$$

Similarly , by applying KVL around second mesh we get,

$$-V_2 - R_3 (I_2 - I_3) - R_4 (I_2 - I_1) = 0$$

$$- V_2 = - I_1 R_4 + I_2 (R_3 + R_4) - I_3 R_3 \dots\dots\dots (2)$$

And by applying KVL around third mesh or loop we get,

$$V_3 - R_1 I_3 - R_3 (I_3 - I_2) - R_2 (I_3 - I_1) = 0$$

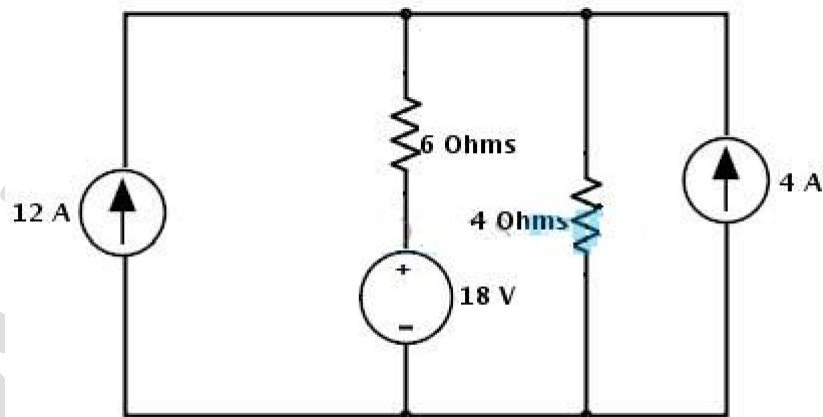
$$V_3 = - I_1 R_2 - I_2 R_3 + I_3 (R_1 + R_2 + R_3) \dots\dots\dots (3)$$

Therefore, by solving the above three equations we can obtain the mesh currents for each mesh in the given circuit.

Example problems on mesh analysis:

Example 1:

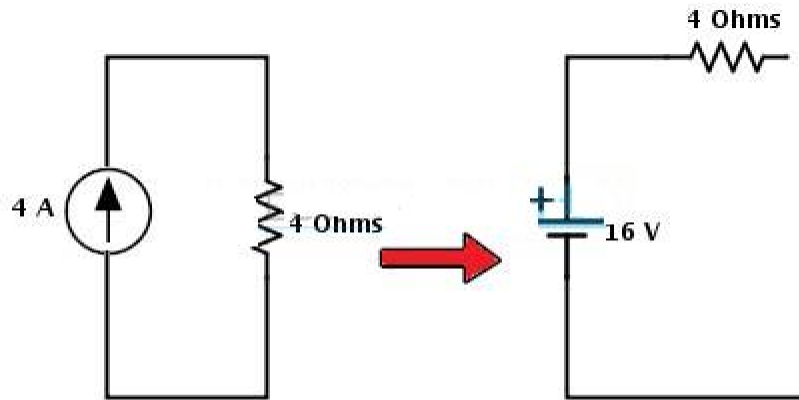
Consider the below example in which we find the voltage across the 12A current source using mesh analysis. In the given circuit all the sources are current sources.



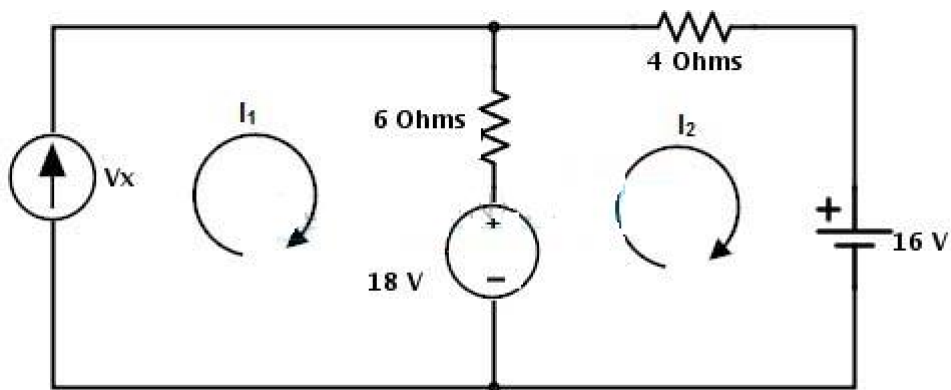
Step 1: In the circuit there is a possibility to change the current source to a voltage source on right hand side source with parallel resistance. The current source is converted into a voltage source by placing the same value of resistor in series with a voltage source and the voltage in that source is determined as

$$V_s = I_s R_s$$

$$= 4 \times 4 = 16 \text{ Volts}$$



Step 2: Assign the branch currents as I_1 and I_2 to the respective branches or loops and represent the direction of currents as shown below.



Step 3: Apply the KVL to each mesh in the given circuit

Mesh -1:

$$V_x - 6 \times (I_1 - I_2) - 18 = 0$$

Substituting $I_1 = 12 \text{ A}$

$$V_x + 6I_2 = 90 \dots\dots\dots (1)$$

Mesh - 2:

$$18 - 6 \times (I_2 - I_1) - 4 \times I_2 - 16 = 0$$

$$2 - 10 \times I_2 + 6(12) = 0$$

$$I_2 = 74/10$$

$$= 7.4 \text{ Amps}$$

Substituting in equation 1 we get

$$V_x = 90 - 44.4$$

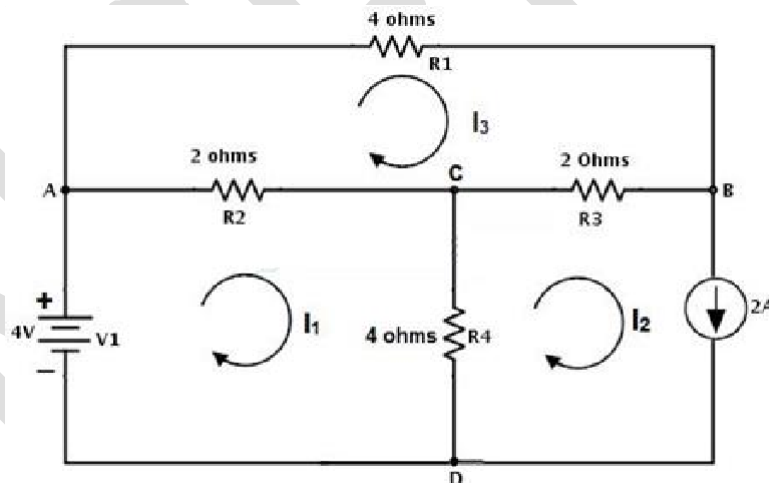
$$= 45.6 \text{ Volts}$$

Example 2:

Consider the below circuit where we determine the voltage across the current source and a branch current I_{ac} . Assign the directions as shown below and note that current is assigned opposite to the source current in second loop.

By applying KVL to the first mesh we get

$$V_1 - R_2 (I_1 - I_3) - R_4 (I_1 - I_2) = 0$$



$$4 - 2 I_1 - 2I_3 - 4I_1 - 4I_2 = 0$$

$$-6I_1 - 2I_3 = 4 \dots\dots\dots(1)$$

By applying KVL to the second mesh we get

$$-V_c - R_4(I_2 - I_1) - R_3 (I_2 - I_3) = 0$$

$$-V_c = 4I_2 - 4I_1 + 2I_2 - 2I_3 = 0$$

$$-V_c = -4I_1 + 6I_2 - 2I_3$$

But $I_2 = -2$ A, then

$$-V_c = -4I_1 - 12 - 2I_3 \dots \dots \dots (2)$$

By applying KVL to the third mesh we get

$$-R_1 I_3 - R_3 (I_3 - I_2) - R_2 (I_3 - I_1) = 0$$

$$-4 I_3 - 2I_3 + 2I_2 - 2I_3 + 2I_1 = 0$$

$$-8I_3 - 4 + 2I_1 = 0 \text{ (by substituting } I_2 = -2 \text{ A)}$$

$$2I_1 - 8I_3 = 4 \dots \dots \dots (3)$$

By solving 1 and 3 equations we get $I_3 = -0.615$ and $I_1 = 4.46$

Therefore, the voltage $V_c = 4(4.46) + 12 + 2(-0.615)$

$$V_c = 28.61 \text{ V}$$

And the branch current $I_{ac} = I_1 - I_3$

$$I_{ac} = 5.075 \text{ amps}$$

Likewise we can find every branch current using the mesh analysis.

Example: 3 Using mesh analysis, obtain the current through the 10V battery for the circuit shown in figure 1.

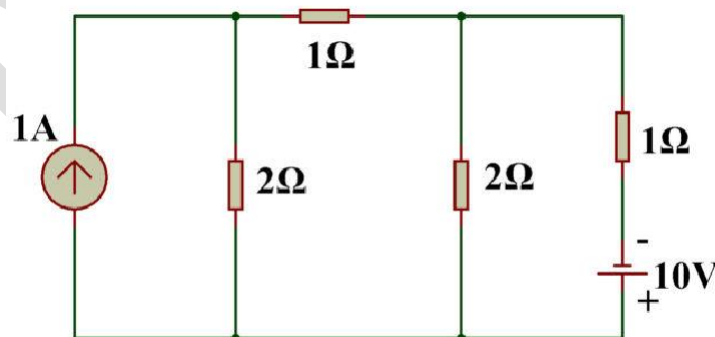


Figure: 1

Solution: The current source is first converted to an equivalent voltage source and the loop currents are named (Figure 2).

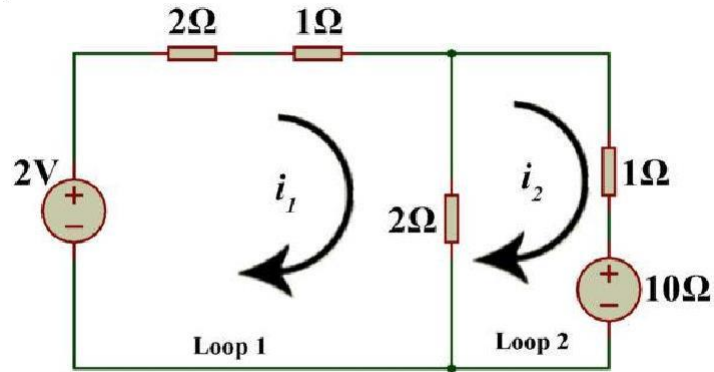


Figure: 2

Applying KVL in loop-1,

$$-2 + 3i_1 + (i_1 - i_2)2 = 0$$

$$\text{or, } 5i_1 - 2i_2 = 2 \quad \dots(1)$$

and in loop-2,

$$2(i_2 - i_1) + i_2 - 10 = 0$$

$$\text{or, } -2i_1 + 3i_2 - 10 = 0 \quad \dots(2)$$

Solving equations (1) & (2),

$$i_2 = 4.91A \quad \text{and} \quad i_1 = 2.36A$$

∴ Current of the 10V battery is 4.91A which enters the battery through -ve terminal as shown.

Example: 4 In figure 5, obtain the mesh equations. If $e_1 = e_2 = e_3 = 1V$ and all resistances are equal to each other, being 1Ω each, what would be the loop currents?

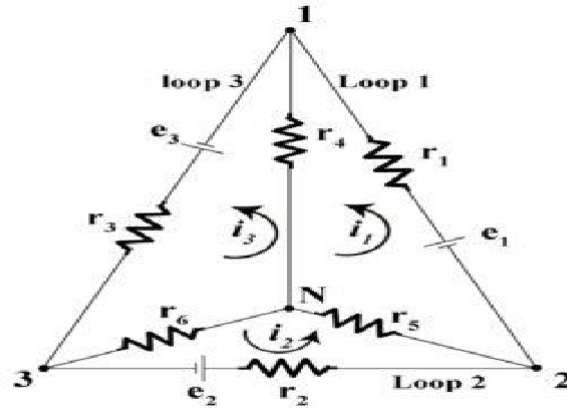


Figure: 5

Solution:

Let the loop currents be i_1 , i_2 , and i_3 .

The loop equations are given below:

For loop-1

$$i_1 r_1 + (i_1 - i_2 - i_3) r_4 + (i_1 - i_2) r_5 = e_1$$

$$\text{or, } i_1(r_1 + r_4 + r_5) - i_2 r_5 - i_3 r_4 = e_1 \quad \dots(i)$$

For loop-2

$$(i_2 - i_1) r_5 + (i_2 - i_3) r_6 + i_2 r_2 = e_2$$

$$\text{or, } -i_1 r_5 + i_2(r_2 + r_5 + r_6) - i_3 r_6 = e_2 \quad \dots(ii)$$

For loop-3

$$i_3 r_3 + (i_3 - i_2) r_6 + (i_3 - i_1) r_4 = e_3$$

$$\text{or, } -i_1 r_4 - i_2 r_6 + i_3(r_3 + r_4 + r_6) = e_3 \quad \dots(iii)$$

If $e_1 = e_2 = e_3 = 1V$, and all resistance are equal, from symmetry it is evident that

$$i_1 = i_2 = i_3 = i$$

Thus for loop-1

$$e_1 = 1 = i r = 1 \times i$$

$$\therefore I = 1A$$

The loop currents in all loops will be identical and equal to 1A for each loop

MESH MATRIX METHOD

- R_1, R_2, R_3, R_4 and R_5 are the various resistances
- V_1 and V_2 are the voltage source
- I_1 is the current flowing in the mesh ABFEA
- I_2 is the current flowing in the mesh BCGFB
- I_3 is the current flowing in the mesh CDHGC

The direction of the current is assumed in the clockwise for simplicity in solving the network.

Steps for Solving Network by Mesh Current Method

Considering the above circuit diagram, the following steps are given below to solve the circuit by the Mesh Current method.

Step 1 – First of all, identify the independent circuit meshes or loop. As there is three mesh in the circuit diagram shown above which are considering.

Step 2 – Assign a circulating current to each mesh as shown in circuit diagram where I_1, I_2 and I_3 are flowing in each mesh.

It is preferable to assign the same direction of all the currents and in a clockwise direction for making the calculation easier.

Step 3 – Now, write the KVL equation for each mesh.

As there are three meshes in the circuit, there will be three KVL equations as shown below

Applying KVL in the mesh ABFEA

$$I_1 R_1 + (I_1 - I_2) R_2 = V_1$$

By rearranging the equation, we will get an equation (1)

$$I_1 (R_1 + R_2) + I_2 (-R_2) + I_3 (0) = V_1 \dots \dots \dots (1)$$

Applying KVL in the mesh BCGFB

$$I_2 R_3 + (I_2 - I_3) R_4 + (I_2 - I_1) R_2 = 0 \quad \text{or}$$

$$I_1(-R_2) + I_2(R_2 + R_3 + R_4) + I_3(-R_4) = 0 \dots\dots\dots (2)$$

Applying KVL in the mesh CDHGC

$$I_3 R_5 + V_2 + (I_3 - I_2) R_4 = 0 \quad \text{or}$$

$$I_1(0) + I_2(-R_4) + I_3(R_4 + R_5) = -V_2 \dots\dots\dots (3)$$

Step 4 – Now solve equations (1) (2) and (3) simultaneously to get the value of current I_1 , I_2 and I_3 .

By knowing the mesh currents, we can determine the various voltages and currents in the circuit.

Matrix Form

The above circuit can be solved by the Matrix method also, as shown below

The above equations (1), (2) and (3) in matrix form can be expressed as

$$\begin{bmatrix} R_1 + R_2 & -R_2 & 0 \\ -R_2 & R_2 + R_3 + R_4 & -R_4 \\ 0 & -R_4 & R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ -V_2 \end{bmatrix} \dots\dots\dots (4)$$

Thus, the equation (4) can be solved to get the values of the various currents.

It is seen from the equation (4) that the resistance matrix $[R]$ is symmetric, i.e.

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \pm V_1 \\ \pm V_2 \\ \pm V_3 \end{bmatrix} \dots\dots\dots (5)$$

Equation (5) can be written as:

$$[R][I] = [V]$$

Where,

[**R**] is the mesh resistance

[**I**] is the column vector of mesh currents and

[**V**] is the column vector of the algebraic sum of all the source voltages around the mesh.

NODE ANALYSIS/NODE VOLTAGE METHOD

Definition of Nodal Analysis

Nodal analysis is a method that provides a general procedure for analyzing circuits using node voltages as the circuit variables. **Nodal Analysis** is also called the **Node-Voltage Method**. Some Features of Nodal Analysis are as

- **Nodal Analysis** is based on the application of the Kirchhoff's Current Law (KCL).
- Having n nodes there will be $n-1$ simultaneous equations to solve.
- Solving $n-1$ equations all the nodes voltages can be obtained.
- The number of non reference nodes is equal to the number of Nodal equations that can be obtained.

Types of Nodes in Nodal Analysis

- **Non Reference Node** – It is a node which has a definite Node Voltage. E.g. Here Node 1 and Node 2 are the Non Reference nodes
- **Reference Node** – It is a node which acts a reference point to all the other node. It is also called the Datum Node.

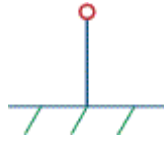
Types of Reference Nodes

1. **Chassis Ground** – This type of reference node acts a common node for more than one circuits.



2. **Earth Ground** – When earth potential is used as a reference in any circuit then this type of

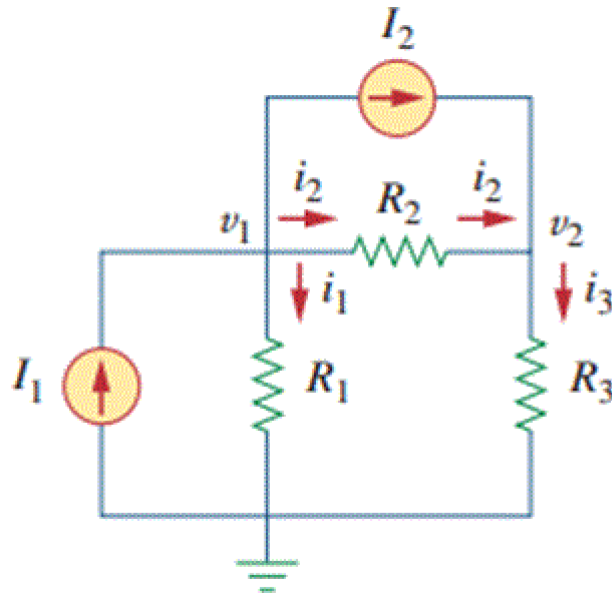
reference node is called Earth Ground.



Solving of Circuit Using Nodal Analysis

Basic Steps Used in Nodal Analysis

1. Select a node as the reference node. Assign voltages V_1, V_2, \dots, V_{n-1} to the remaining nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the non reference nodes.
3. Use Ohm's law to express the branch currents in terms of node voltages.



Node Always assumes that current flows from a higher potential to a lower potential in resistor. Hence, current is expressed as follows

$$I = \frac{V_{high} - V_{low}}{R}$$

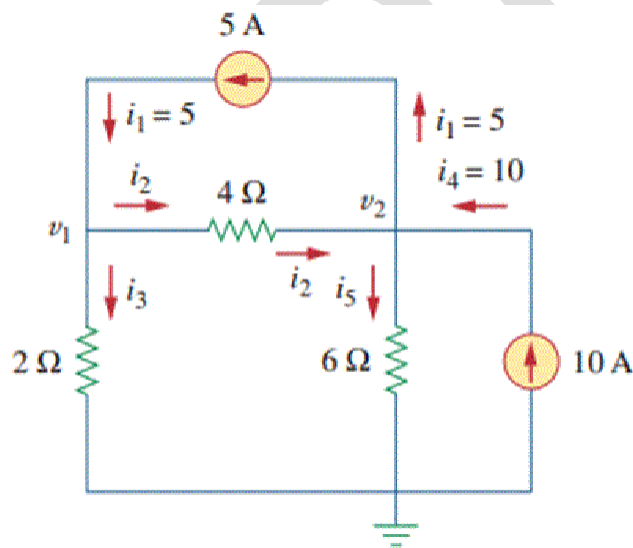
IV. After the application of Ohm's Law get the $(n-1)$ node equations in terms of node voltages and resistances.

V. Solve $(n-1)$ node equations for the values of node voltages and get the required node Voltages as result.

Nodal Analysis with Current Sources

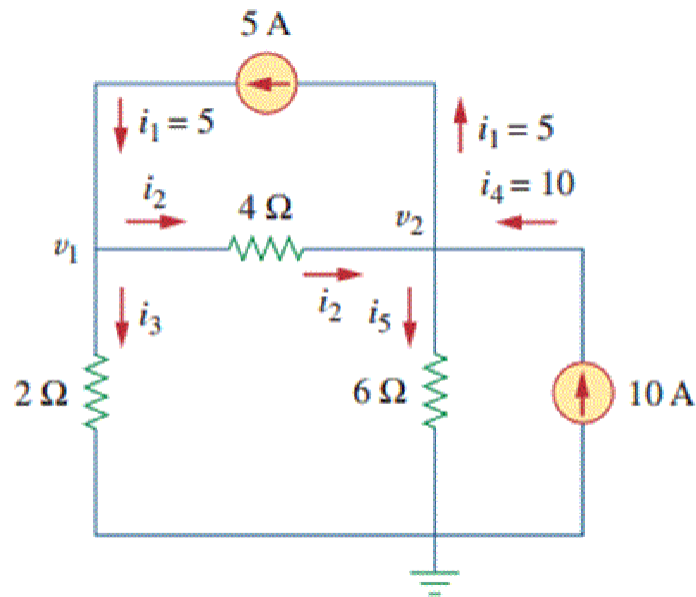
Nodal analysis with current sources is very easy and it is discussed with an example below.

Example: Calculate Node Voltages in following circuit



In the following circuit we have 3 nodes from which one is reference node and other two are non reference nodes – Node 1 and Node 2.

Step I. Assign the nodes voltages as v_1 and v_2 and also mark the directions of branch currents with respect to the reference nodes



Step II. Apply KCL to Nodes 1 and 2

KCL at Node 1

$$i_1 = i_2 + i_3 \dots\dots(1)$$

KCL at Node 2

$$i_2 + i_4 = i_1 + i_5 \dots\dots(2)$$

Step III. Apply Ohm's Law to KCL equations

• Ohm's law to KCL equation at Node1

$$i_1 = i_2 + i_3 \Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Simplifying the above equation we get,

$$3v_1 - v_2 = 20 \dots\dots(3)$$

• Now, Ohm's Law to KCL equation at Node 2

$$i_2 + i_4 = i_1 + i_5 \Rightarrow \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

Simplifying the above equation we get

$$-3v_1 + 5v_2 = 60 \dots\dots(4)$$

Step IV. Now solve the equations 3 and 4 to get the values of v_1 and v_2 as,
Using elimination method

$$\begin{aligned} 3v_1 - v_2 &= 20 \\ -3v_1 + 5v_2 &= 60 \\ \Rightarrow 4v_2 &= 80 \Rightarrow v_2 = 20 \text{ Volts} \end{aligned}$$

And substituting value $v_2 = 20$ Volts in equation (3) we get-

$$3v_1 - 20 = 20 \Rightarrow v_1 = \frac{40}{3} = 13.333 \text{ Volts}$$

Hence node voltages are as $v_1 = 13.33$ Volts and $v_2 = 20$ Volts.

Nodal Analysis with Voltage Sources

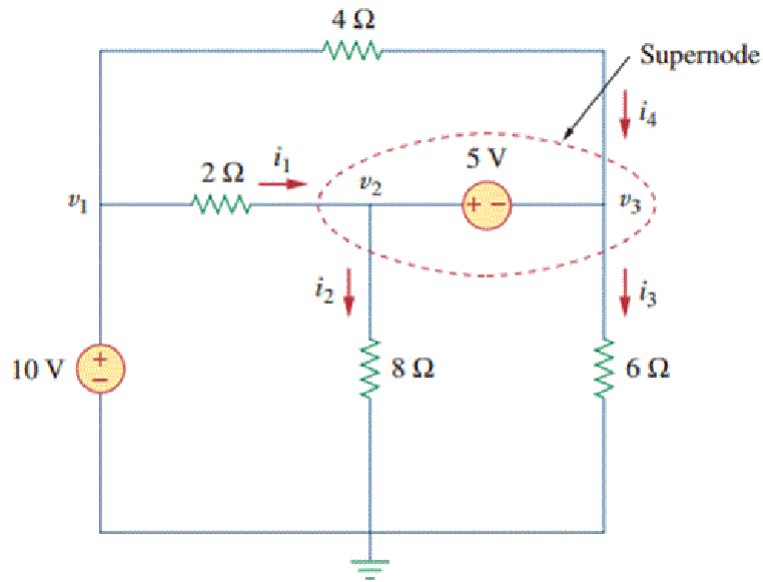
Case I. If a voltage source is connected between the reference node and a non reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source and its analysis can be done as we done with current sources. $V_1 = 10$ Volts.

Case II. If the voltage source is between the two non reference nodes then it forms a supernode whose analysis is done as following

Supernode Analysis

Definition of Super Node

Whenever a voltage source (Independent or Dependent) is connected between the two non reference nodes then these two nodes form a generalized node called the Super node. So, Super node can be regarded as a surface enclosing the voltage source and its two nodes.



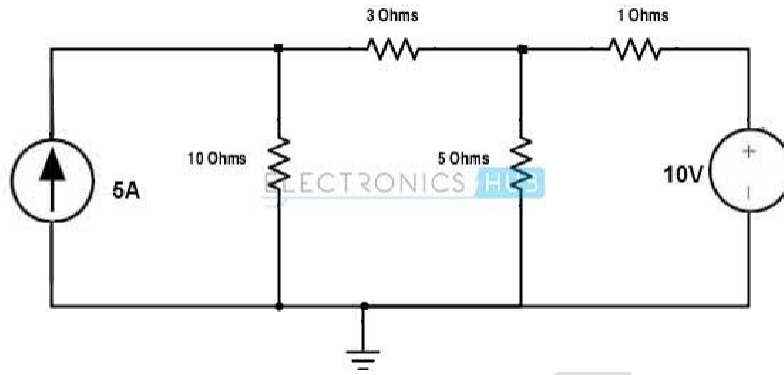
In the above Figure 5V source is connected between two non reference nodes Node – 2 and Node – 3. So here Node – 2 and Node – 3 form the Super node.

Properties of Supernode

- Always the difference between the voltage of two non reference nodes is known at Supernode.
- A supernode has no voltage of its own
- A supernode requires application of both KCL and KVL to solve it.
- Any element can be connected in parallel with the voltage source forming the supernode.
- A Supernode satisfies the KCL as like a simple node.

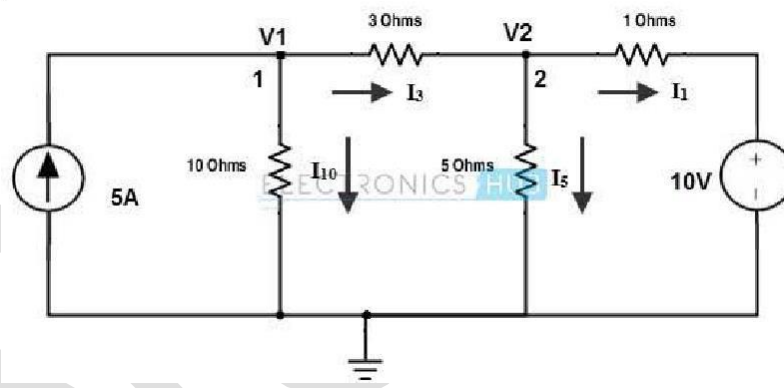
Example 1

Determine the node voltages and currents in each branch using nodal analysis method in the given circuit.



The given circuit contains a voltage source. This can be transformed to current source or can be analysed directly without any transformation. Now let us calculate the nodal voltages without any transformation.

As a first step in nodal analysis, we have to choose and label the nodes present in the given circuit. By choosing the bottom node as reference node, we have two other nodes in the given circuit. So these nodes are labeled as V1 and V2 as shown in below figure. And also current directions in each branch are represented.



By applying KCL at node 1, we get

$$5 = I_3 + I_{10}$$

$$5 = (V_1/10) + (V_1 - V_2/3)$$

$$13V_1 - 10V_2 = 150 \dots\dots\dots(1)$$

By applying KCL at node 2, we get

$$I_3 = I_5 + I_1$$

$$(V_1 - V_2/3) = (V_2/5) + (V_2 - 10/1)$$

$$5V_1 - 23V_2 = -150 \dots\dots\dots (2)$$

By solving above two equations, we get

$$V_1 = 19.85 \text{ Volts and } V_2 = 10.9 \text{ Volts}$$

The currents in each branch is given as

$$\begin{aligned} I_{10} &= V_1/10 \\ &= 19.85/10 = 1.985 \end{aligned}$$

$$\begin{aligned} I_3 &= V_1 - V_2/3 \\ &= 19.85 - 10.9/3 \\ &= 2.98 \text{ A} \end{aligned}$$

$$\begin{aligned} I_5 &= V_2/5 \\ &= 10.9/5 \\ &= 2.18 \text{ A} \end{aligned}$$

$$\begin{aligned} I_1 &= V_2 - 10 \\ &= 10.9 - 10 \\ &= 0.9 \text{ A} \end{aligned}$$

Example 2: Using Nodal method, find the current through resistor r_2 (Figure 1).

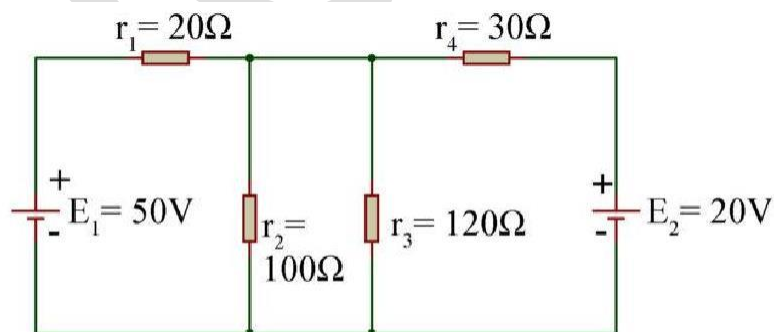


Figure: 1

Solution: Let us redraw the circuit with naming of the nodes and branch current as shown in figure 2.

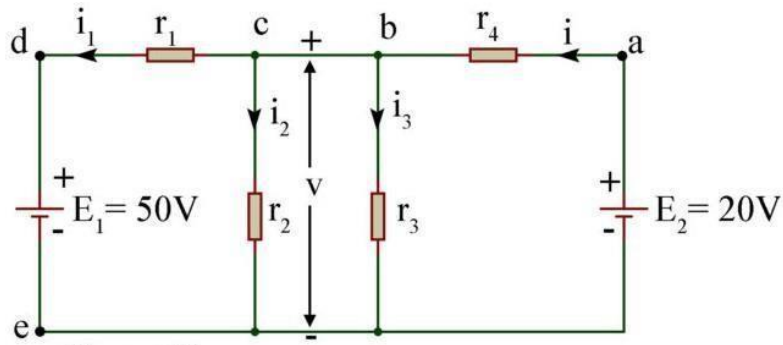


Figure: 2

At node -b, $i = i_1 + i_2 + i_3$ (electrically nodes b and c are same)

Assuming the polarity of the voltage v at node c or b, we thus get.

$$\frac{20 - v}{r_4} = \frac{v - 50}{r_1} + \frac{v}{r_2} + \frac{v}{r_3}$$

or,
$$\frac{v - 20}{30} + \frac{v - 50}{20} + \frac{v}{100} + \frac{v}{120} = 0$$

$$\therefore v = 31.18V$$

$$\therefore i_2 = \frac{v}{r_2} = \frac{31.18}{100} A = 0.3118A$$

i.e. current through $r_2 = 311.8mA$.

Example 4: In the circuit of figure 7, find the current in 1Ω resistor.

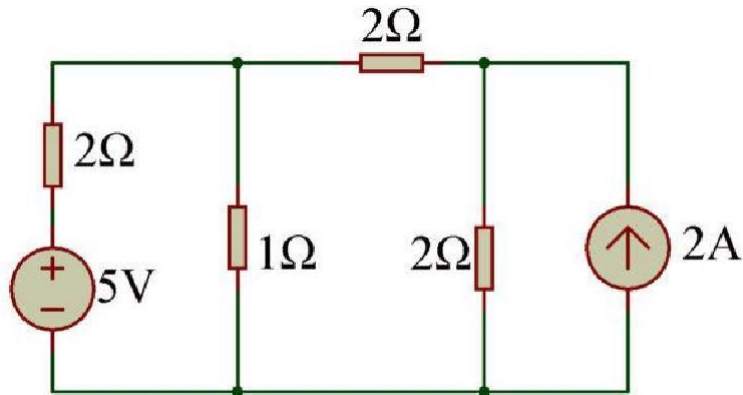


Figure: 7

Solution:

Let us first convert the current source of figure 7 to voltage source and draw the equivalent network (figure 8). Let the +ve voltage at node (1) be v_1 V.

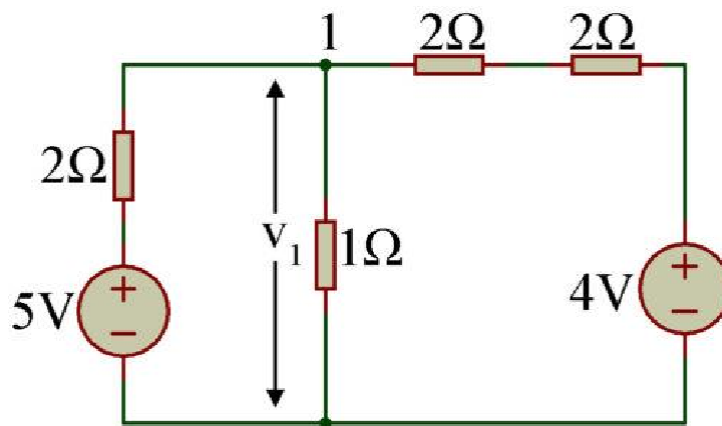


Figure: 8

∴ Using nodal analysis,

$$\frac{v_1}{1} + \frac{v_1 - 5}{2} + \frac{v_1 - 4}{4} = 0$$

or, $v_1 = 2V$

Hence, the current through 1Ω resistor is

$$\frac{v_1}{1} = 2A$$

MODULE II

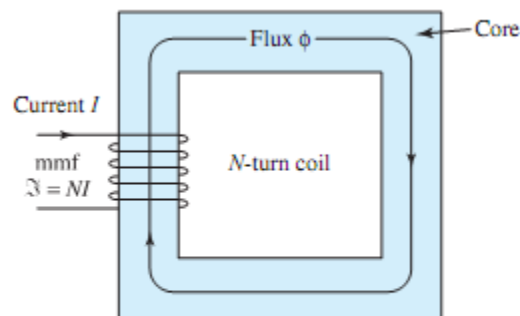
ELEMENTARY CONCEPTS OF MAGNETIC CIRCUITS, ELECTROMAGNETIC INDUCTION AND AC FUNDAMENTALS

INTRODUCTION TO MAGNETIC CIRCUITS

Definition

The closed path followed by the magnetic flux is magnetic circuit. All electric power machinery such as **generator**, **transformer**, **motors** depend for their operation on the magnetic circuits.

A magnetic circuit consists of a structure composed for the most of high permeability magnetic material. The core is assumed to be composed of magnetic material whose permeability is much greater than that of the surrounding air. The core is of uniform cross-section and is excited by a winding having N turns and carrying a current of I amperes. This winding develops a magnetic field in the core. The magnetic field is in terms of flux lines which form closed loops interlinking with the winding.



Simple Magnetic Circuit

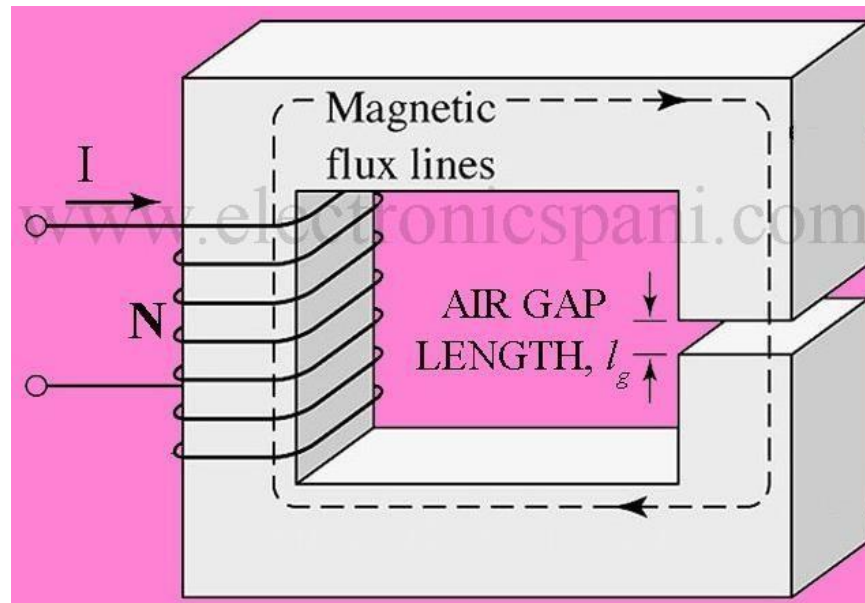
Magnetic Circuit Basics

A closed path followed by magnetic flux is known as a **magnetic circuit**. In a magnetic circuit, flux starts from one point and finishes at the same point. A magnetic circuit usually consists of magnetic materials having high permeability such as iron, soft-steel, etc. since they offer small opposition to magnetic flux.

Magnetic Circuits with Air-Gaps.

Energy-conversion devices which incorporate a moving element have necessarily air gaps in their magnetic circuits. Air-gaps are also provided in the magnetic circuits to avoid Saturation. A

magnetic circuit with an air gap is shown in Fig. 2. An air-gap is nothing else but a volume of air between two magnetic surfaces. The length of air gap l_g equals the distance between the two magnetic surfaces. The area of x-section of any one of the surfaces gives the air-gap area a_g . When the air-gap length l_g is much smaller than the dimensions of the adjacent core faces, the magnetic flux ϕ is constrained essentially to reside in the core and the air gap and is continuous throughout the magnetic circuit.



magnetic circuit with air gap

Thus, the configuration shown in Fig. 2 can be analyzed as a magnetic circuit with two series components, a magnetic or iron core of permeability μ and mean length l_i and an air-gap of permeability μ_0 and length l_g . Since the permeability of air is constant, the air-gap is a linear part of the magnetic circuit and the flux density in the air-gap is proportional to the mmf across the air-gap. The necessary mmf is calculated separately for the air-gap and the iron portions and then added to determine the total mmf.

The magnetic flux is usually produced by an electric current through a solenoid (having a large number of turns). There are three types of magnetic circuits:

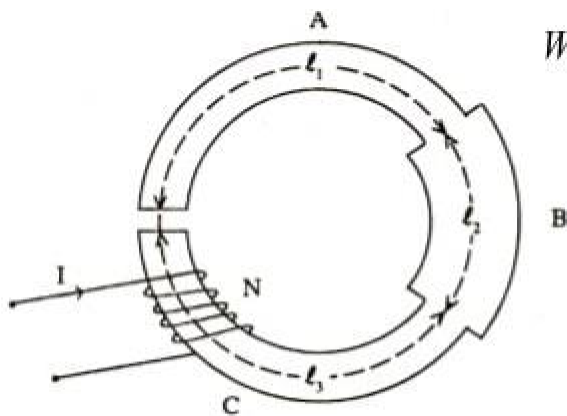
- Series Magnetic Circuit
- Parallel Magnetic Circuit
- Series-parallel Magnetic Circuits

SERIES MAGNETIC CIRCUIT

A magnetic circuit that has many parts of different dimensions and materials connected in series is called a series magnetic circuit.

Consider a composite magnetic circuit consisting of three parts having different lengths, area of cross-sections and relative permeability's. It also has an air-gap.

In this case, the reluctance of each part will be different depending upon the dimensions and relative permeability's of that part. The total reluctance will be the sum of the reluctance of individual parts. The same flux will flow through complete circuits.



$$\text{Total Reluctance } S = S_1 + S_2 + S_3 + S_g$$

$$\text{Where } S_1 = \frac{l_1}{\mu_0 \mu_{r1} a_1}$$

$$S_2 = \frac{l_2}{\mu_0 \mu_{r2} a_2}$$

$$S_3 = \frac{l_3}{\mu_0 \mu_{r3} a_3}$$

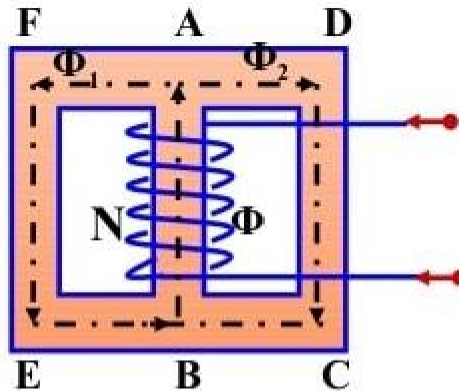
$$S_g = \frac{l_g}{\mu_0 \mu_{rg} a_g}$$

$$\text{mmf} = (\phi_1 S_1 + \phi_2 S_2 + \phi_3 S_3 + \phi_g S_g) = \left(\frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_3 l_3}{\mu_0 \mu_{r3}} + \frac{B_g l_g}{\mu_0 \mu_{rg}} \right)$$

$$\text{Total mmf} = H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

PARALLEL MAGNETIC CIRCUIT

A magnetic circuit that has more than one path for the magnetic flux is called a **parallel magnetic circuit**. It is similar to a parallel electric circuit that has more than one for electric current to flow.



$$S_{AB} = \frac{l}{\mu_0 \mu_{r1} a}; S_{ADCB} = \frac{l_2}{\mu_0 \mu_{r2} a_2}; S_{AFEB} = \frac{l_1}{\mu_0 \mu_{r3} a_3}; \phi = \phi_1 + \phi_2$$

Total mmf required = Path AB mmf + (Path AFEB or ADCB mmf)

$$MMF_{Total} = \phi S_{AB} + (\phi_1 S_{AFEB} \text{ or } \phi_2 S_{ADCB}),$$

$$\text{Where } \phi_1 S_{AFEB} = \phi_2 S_{ADCB}$$

Consider a parallel magnetic circuit shown in Figure. A current-carrying coil is wound on the central limb. The flux set up by this coil is divided at A into two paths i.e.

- Flux ϕ_1 passes along the path AFEB,
- Flux ϕ_2 passes along the path ADCB

It is clear that $\phi = \phi_1 + \phi_2$

Here the two magnetic paths AFEB and ADCB are in parallel, the total MMF required for this parallel circuit will be equal to the MMF required for any one of the above said paths.

MAGNETIC FLUX DENSITY (B)

The flux per unit area is defined as the *magnetic flux density*. It is measured in a plane perpendicular to flux.

$$\text{Magnetic Flux Density, } \mathbf{B} = \phi \div \mathbf{A}$$

Units: Weber per meter square (Wb/m^2) or **tesla(T)**.

MAGNETIC FIELD INTENSITY (H)

The magnetic field strength or magnetic field intensity is given by MMF per unit length of the magnetic circuit.

Magnetic Field Intensity, $H = (NI) \div l$

where N = Number of turns of magnetizing coil

I = Current through the coil

l = length of magnetic material in meters

Units: **AT/m**.

The magnetic field intensity is also known as magnetic field strength or magnetizing force.

PERMEABILITY (μ)

The ability of a material to carry the magnetic lines of flux is known as *permeability* of that material.

The magnetic lines of force can pass through high permeability materials like iron, steel, very easily. Low permeability materials like wood etc. don't allow the flux lines to pass through them easily.

Absolute Permeability

It is the ratio of flux density (B) in a particular medium to the magnetic field strength (H) which produces magnetic flux density. It is denoted by μ .

Absolute Permeability, $\mu = \mu_0 \mu_r$

Units: Henry/meter (**H/m**)

PERMEABILITY OF THE AIR/SPACE/VACUUM(μ_0)

If a magnet is kept in air or vacuum, then the ratio of flux density (B) and magnetic field strength (H) is defined as the permeability of free space. It is denoted by μ_0 .

Permeability of Free Space, $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$

RELATIVE PERMEABILITY (μ_R)

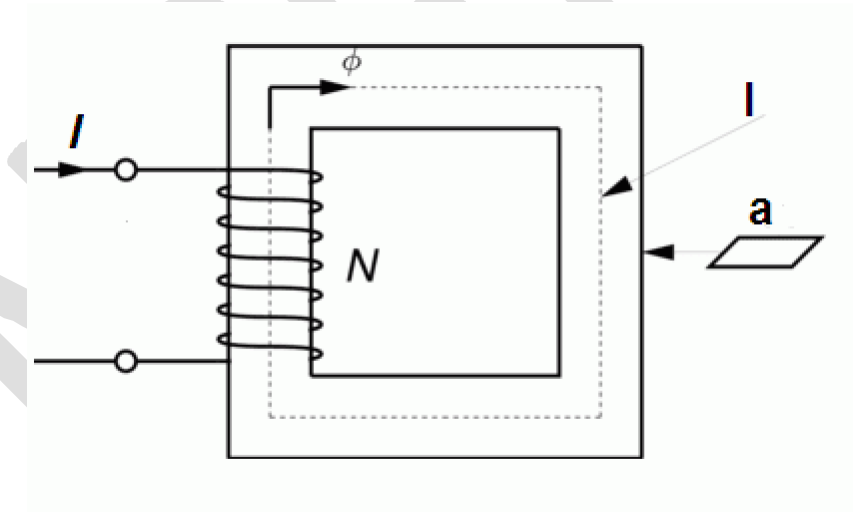
The ratio of permeability of material to the permeability of vacuum or air is known as *relative permeability*.

Relative Permeability, $\mu_r = \mu \div \mu_0$

It has no units .

The relative permeability of vacuum, air and all non-magnetic materials is 1. The relative permeability of all the magnetic materials is very high. For example, the relative permeability of permalloy (nickel 78% and iron 22%) is about 50000.

MAGNETO-MOTIVE FORCE (MMF)



The *magneto-motive force* is the driving force which produces the magnetic flux. The magnetic field intensity (H) is decided by MMF.

Magneto-motive Force, $\text{MMF} = NI$

where N = Number of turns of magnetizing coil

I = Current through the coil

Units: Ampere Turns (**AT**)

RELUCTANCE (S)

It is opposition offered to the flow of magnetic flux by the magnetic material.

Unit: **AT/Wb**

Reluctance, $S = l \div (\mu \times a)$

where l = length of the magnetic path in meters.

a = area of the cross-section of magnetic path in meter square.

μ = absolute permeability of medium in H/m.

$$= \mu_0 \mu_r$$

Therefore, Reluctance, $S = l \div (\mu_0 \mu_r a)$

The reluctance is also given by the ratio of the MMF and the amount of flux produced.

i.e. Reluctance, $S = \text{MMF} \div \text{flux}$

Reluctance, $S = (NI) / \phi$

PERMEANCE

The *permeance* of a material represents the ease with which magnetic flux can be produced in that material. It is reciprocal of reluctance. Its unit is Wb/AT or henry.

MAGNETIC SUSCEPTIBILITY

The magnetization of a material is proportional to the field and the proportionality constant is called the magnetic susceptibility. In other words, it can be defined as the ratio of magnetization **M** to magnetizing force **H** i.e.

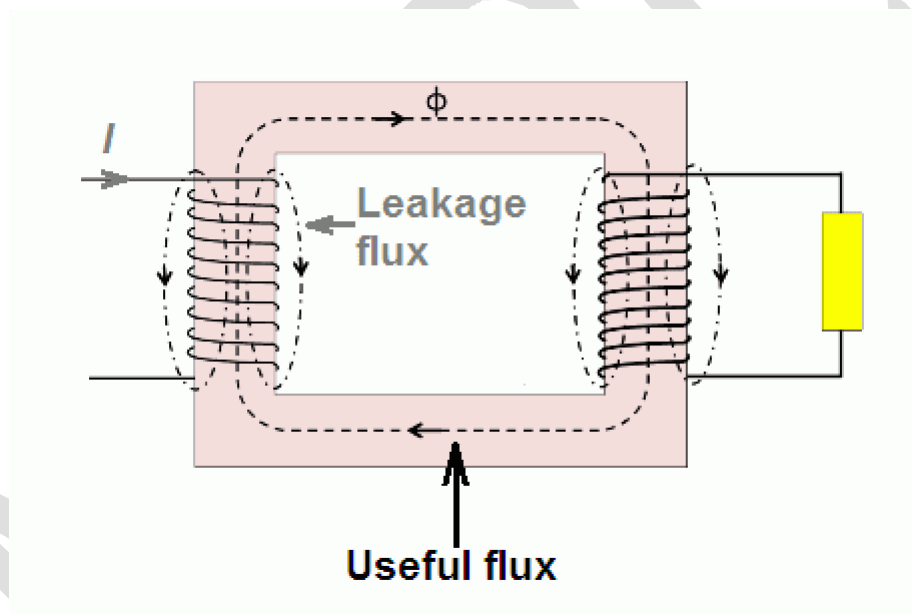
Magnetic Susceptibility, $X_m = M/H$

It is a dimensionless quantity. It is a measure of how easily a material is magnetized in a magnetizing field. Its value for vacuum is zero as there can be no magnetization in the vacuum.

We can classify materials in terms of \mathbf{X}_m . Materials with positive values of \mathbf{X}_m are paramagnetic and those with negative values of \mathbf{X}_m are diamagnetic. For ferromagnetic materials, \mathbf{X}_m is positive and very large.

LEAKAGE FLUX

The part of the total magnetic flux which flows through the magnetic circuit is called useful magnetic flux. However, the magnetic flux which does not completely pass through the magnetic path, but partially passes through the air is called *leakage magnetic flux*.



Mathematically, $\phi_{\text{total}} = \phi_{\text{useful}} + \phi_{\text{leakage}}$

Leakage Factor (λ)

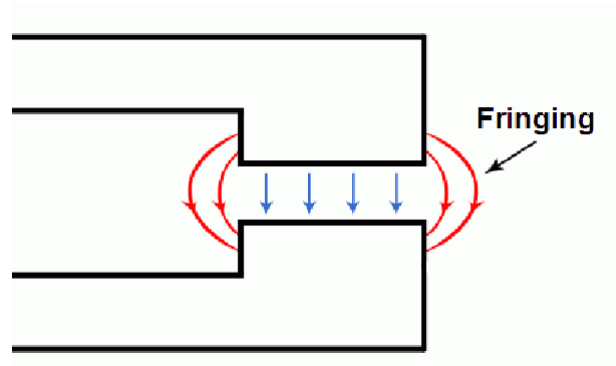
The ratio of total flux produced to the useful flux is called leakage factor or leakage coefficient.

Leakage factor, $\lambda = \phi_{\text{total}} / \phi_{\text{useful}}$

The value of *leakage factor* is always greater than unity. Typical values of leakage factor are from 1.12 to 1.25. In the magnetic circuits, the magnetic leakage can be minimized by placing the exciting coils as close as possible to the points where the flux is to be utilized.

FRINGING

The magnetic lines of force repel each other while passing through a non-magnetic material. Due to this when the flux lines cross the air gap, they tend to bulge outwards. This effect is known as *fringing*.



The effect of fringing is to make the effective air gap area larger than that of magnetic path and consequently, the flux density in the air gap is reduced. The effect of fringing depends upon the length of the air gap. To minimize fringing, the air gap length is kept as small as possible. The effect of fringing can be neglected if air gap length is very small as compared to its width.

EXAMPLE PROBLEMS

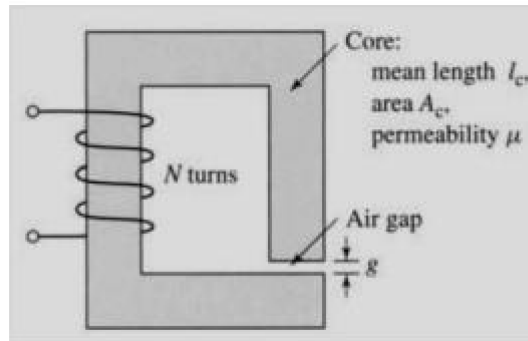
1. A magnetic circuit with a single air gap is shown in Fig. 1.24. The core dimensions are:

$$\text{Cross-sectional area } A_c = 1.8 \times 10^{-3} \text{ m}^2$$

$$\text{Mean core length } l_c = 0.6 \text{ m}$$

$$\text{Gap length } g = 2.3 \times 10^{-3} \text{ m}$$

$$N = 83 \text{ turns}$$



Assume that the core is of infinite permeability ($\mu \rightarrow \infty$) and neglect the effects of fringing fields at the air gap and leakage flux. (a) Calculate the reluctance of the core, R_c and that of the gap R_g . For a current of $i = 1.5$ A, calculate (b) the total flux ϕ , (c) the flux linkages λ of the coil, and (d) the coil inductance L .

Solution:

$$R_c = 0 \quad \text{since } \mu \rightarrow \infty \qquad R_g = \frac{g}{\mu_0 A_c} = \frac{2.3 \times 10^{-3}}{4\pi \times 10^{-7} \times 1.8 \times 10^{-3}} = 1.017 \times 10^6 \text{ A/Wb}$$

$$\phi = \frac{Ni}{R_c + R_g} = \frac{83 \times 1.5}{1.017 \times 10^6} = 1.224 \times 10^{-4} \text{ Wb}$$

$$\lambda = N\phi = 1.016 \times 10^{-2} \text{ Wb}$$

$$L = \frac{\lambda}{i} = \frac{1.016 \times 10^{-2}}{1.5} = 6.773 \text{ mH}$$

Solution:

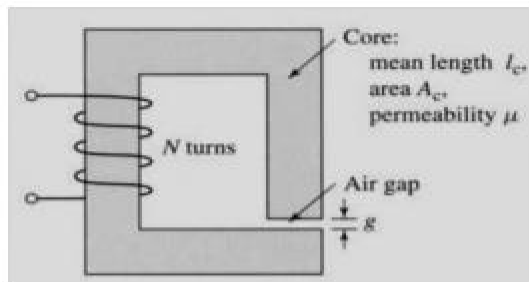
$$R_c = 0 \quad \text{since } \mu \rightarrow \infty \qquad R_g = \frac{g}{\mu_0 A_c} = \frac{2.3 \times 10^{-3}}{4\pi \times 10^{-7} \times 1.8 \times 10^{-3}} = 1.017 \times 10^6 \text{ A/Wb}$$

$$\phi = \frac{Ni}{R_c + R_g} = \frac{83 \times 1.5}{1.017 \times 10^6} = 1.224 \times 10^{-4} \text{ Wb}$$

$$\lambda = N\phi = 1.016 \times 10^{-2} \text{ Wb}$$

$$L = \frac{\lambda}{i} = \frac{1.016 \times 10^{-2}}{1.5} = 6.773 \text{ mH}$$

2. Consider the magnetic circuit of with the dimensions of Problem 1.1. Assuming infinite core permeability, calculate (a) the number of turns required to achieve an inductance of 12 mH and (b) the inductor current which will result in a core flux density of 1.0 T.



Solution:

$$L = \frac{N^2}{R_g} = 12 \times 10^{-3} \text{ mH} \Rightarrow N = \sqrt{12 \times 10^{-3} \times 1.017 \times 10^6} = 110.47 \Rightarrow N = 110 \text{ turns}$$

$$B_c = B_g = 1.0 \text{ T} \Rightarrow \phi = B_g A_c = 1.8 \times 10^{-3} \text{ Wb}$$

$$i = \frac{\lambda}{L} = \frac{N\phi}{L} = \frac{110 \times 1.8 \times 10^{-3}}{12 \times 10^{-3}} = 16.5 \text{ A}$$

ELECTRIC CIRCUIT VS MAGNETIC CIRCUITS

The Difference between the Magnetic and Electric Circuit are explained considering various factors like the basic definition, relation between Flux and Current, Reluctance and Resistance, EMF and MMF, different analogies of both the circuits. Like its density and intensity, laws

applicable in the circuit, Magnetic and Electric lines, etc.

The Difference between Both the circuits are explained below in the tabulated form.

BASIS	MAGNETIC CIRCUIT	ELECTRIC CIRCUIT
Definition	The closed path for magnetic flux is called magnetic circuit.	The closed path for electric current is called electric circuit.
Relation Between Flux and Current	Flux = mmf/reluctance	Current = emf/ resistance
Units	Flux ϕ is measured in weber (wb)	Current I is measured in amperes
MMF and EMF	Magnetomotive force is the driving force and is measured in Ampere turns (AT) $Mmf = \int H \cdot dl$	Electromotive force is the driving force and measured in volts (V) $Emf = \int E \cdot dl$
Reluctance and Resistance	Reluctance opposes the flow of magnetic flux $S = l/\mu$ and measured in (AT/wb)	Resistance opposes the flow of current $R = \rho \cdot l/a$ and measured in (Ω)
Relation between Permeance and Conduction	Permeance = 1/reluctance	Conduction = 1/ resistance
Analogy	Permeability	Conductivity
Analogy	Reluctivity	Resistivity
Density	Flux density $B = \phi/a$ (wb/m ²)	Current density $J = I/a$ (A/m ²)

BASIS	MAGNETIC CIRCUIT	ELECTRIC CIRCUIT
Intensity	Magnetic intensity $H = NI/l$	Electric density $E = V/d$
Drops	Mmf drop = ϕS	Voltage drop = IR
Flux and Electrons	In magnetic circuit molecular poles are aligned. The flux does not flow, but sets up in the magnetic circuit.	In electric circuit electric current flows in the form of electrons.
Examples	For magnetic flux, there is no perfect insulator. It can set up even in the non magnetic materials like air, rubber, glass etc.	For electric circuit there are a large number of perfect insulators like glass, air, rubber, PVC and synthetic resin which do not allow it to flow through them.
Variation of Reluctance and Resistance	The reluctance (S) of a magnetic circuit is not constant rather it varies with the value of B .	The resistance (R) of an electric circuit is almost constant as its value depends upon the value of ρ . The value of ρ and R can change slightly if the change in temperature takes place
Energy in the circuit	Once the magnetic flux sets up in a magnetic circuit, no energy is expanded. Only a small amount of energy is required at the initial stage to create flux in the circuit.	Energy is expanding continuously, as long as the current flows through the electrical circuit. This energy is dissipated in the form of heat.
Applicable Laws	Khirschhoff flux and mmf law is followed	Khirschhoff voltage and current law is followed. (KVL and KCL)
Magnetic and Electric lines	Magnetic lines of flux starts from North pole and ends at South pole.	Electric lines or current starts from positive charge and ends on negative charge.

Magnetic Circuit

The closed path followed by magnetic lines of forces or we can say magnetic flux is called magnetic circuit. A magnetic circuit is made up of magnetic materials having high permeability such as iron, soft steel, etc. Magnetic circuits are used in various devices like electric motor, transformers, relays, generators galvanometer, etc.

Electric Circuit

The rearrangement by which various electrical sources like AC source or DC source, resistances, capacitance and another electrical parameter are connected is called electric circuit or electrical network.

Key Differences Between Magnetic and Electric Circuit

1. The closed path followed by the flux in the Magnetic Circuit, whereas in the Electric Circuit Current follows the closed path.
2. The unit of flux is Weber, and the unit of current is Ampere.
3. Magnetomotive force in the magnetic circuit is the driving force and is measured in Ampere-turns (AT). Electromotive force is the driving force in the electric circuit and is measured in volts (V).
4. Reluctance opposed the flow of magnetic flux $S = l/\mu a$ and measured in (AT/wb) and Resistance opposes the flow of current $R = \rho. l/a$ and measured in (Ω).
5. In the magnetic circuit Permeance = $1/\text{reluctance}$ whereas in the electric circuit Conduction = $1/\text{resistance}$.
6. As in the magnetic circuit, there exists Permeability so as Conductivity in the electric circuit. Similarly, Reluctivity in magnetic circuit is known as Resistivity in the electric circuit.
7. In the magnetic circuit, molecular poles are aligned. The flux does not flow but sets up in the magnetic circuit. In electric circuit electric current flows in the form of electrons.
8. For magnetic flux, there is no perfect insulator. It can set up even in the non-magnetic materials like air, rubber, glass, etc. For electric circuit, there are a large number of perfect insulators like glass, air, rubber, PVC and synthetic resin which do not allow it to flow through them.
9. The reluctance (S) of a magnetic circuit is not constant rather it varies with the value of

B. The resistance (R) of an electric circuit is almost constant as its value depends upon the value of ρ . The value of ρ and R can change slightly if the change in temperature takes place.

10. Once the magnetic flux sets up in a magnetic circuit, no energy is expended. Only a small amount of energy is required at the initial stage to create flux in the circuit. Energy is expanding continuously, as long as the current flows through the electrical circuit. This energy is dissipated in the form of heat.
11. Kirchhoff flux and MMF law is followed in the magnetic circuit whereas in the electric circuit Kirchhoff voltage and current law is followed. (KVL and KCL).
12. Magnetic lines of flux start from The North Pole and ends at the South Pole. Electric lines or current starts from the positive charge and ends on the negative charge.

ELECTROMAGNETIC INDUCTION

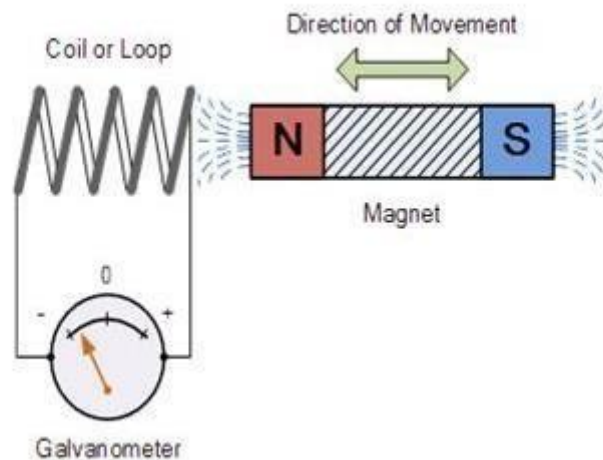
What is Electromagnetic Induction?

Electromagnetic Induction was **discovered by Michael Faraday** in 1831 and James Clerk Maxwell mathematically described it as Faraday's law of induction.

Electromagnetic Induction is a current produced because of voltage production (electromotive force) due to a changing magnetic field.

This either happens when a conductor is placed in a moving magnetic field (when using AC power source) or when a conductor is constantly moving in a stationary magnetic field.

Michael Faraday arranged a conducting wire as per the setup given below, attached to a device to measure the voltage across the circuit. When a bar magnet was moved through the coiling, the voltage detector measures the voltage in the circuit.



Through his experiment, he discovered that there are certain factors that influence this voltage production. They are:

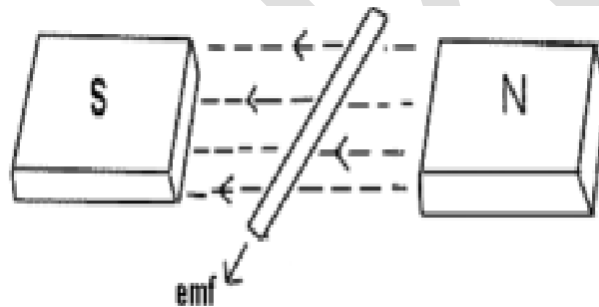
1. **Number of Coils:** The induced voltage is directly proportional to the number of turns/coils of the wire. Greater the number of turns, greater is voltage produced
2. **Changing Magnetic Field:** Changing magnetic field affects the induced voltage. This can be done by either moving the magnetic field around the conductor or moving the conductor in the magnetic field.

The induction of an electromotive force by the motion of a conductor across a magnetic field or by a change in magnetic flux in a magnetic field is called '**Electromagnetic Induction**'.

This either happens when a conductor is set in a moving magnetic field (when utilizing AC power source) or when a conductor is always moving in a stationary magnetic field.

This law of electromagnetic induction was found by **Michael Faraday**. He organized a leading wire according to the setup given underneath, connected to a gadget to gauge the voltage over the circuit. So when a bar magnet passes through the snaking, the voltage is measured in the circuit. The importance of this is a way of producing electrical energy in a circuit by using magnetic fields and not just batteries anymore. The machines like generators, transformers also the motors work on the principle of electromagnetic induction.

Faraday's law of Electromagnetic Induction



- **First law:** Whenever a conductor is placed in a varying magnetic field, EMF induces and this emf is called an induced emf and if the conductor is a closed circuit than the induced current flows through it.
- **Second law:** The magnitude of the induced EMF is equal to the rate of change of flux linkages.

Based on his experiments we now have Faraday's law of electromagnetic induction according to which the amount of voltage induced in a coil is proportional to the number of turns and the changing magnetic field of the coil.

So now, the induced voltage is as follows:

$$e = N \times d\Phi dt$$

where,

e is the induced voltage

N is the number of turns in the coil

Φ is the magnetic flux

t is the time

Lenz's law of Electromagnetic Induction

Lenz law of electromagnetic induction states that, when an emf induces according to Faraday's law, the polarity (direction) of that induced emf is such that it opposes the cause of its production.

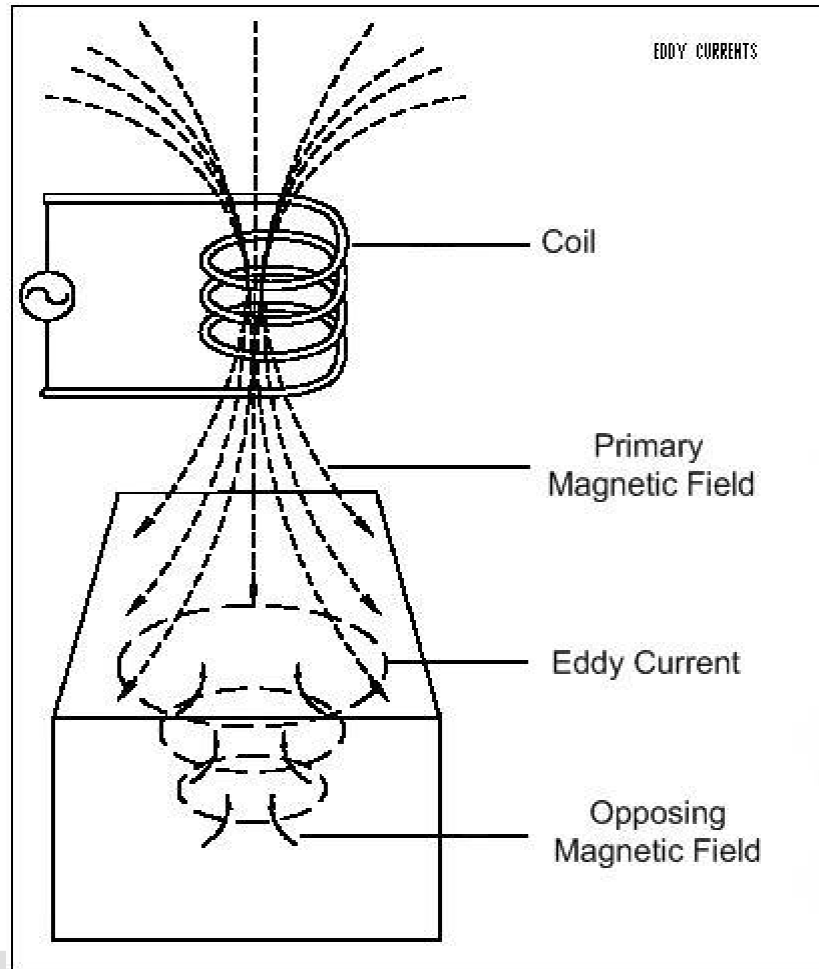
According to Lenz's law

$$E = -N (d\Phi/ dt) \text{ (volts)}$$

Eddy currents

By Lenz law of electromagnetic induction, the current swirls in such a way as to create a magnetic field opposing the change. Because of the tendency of eddy currents to oppose, eddy currents cause a loss of energy. Eddy currents transform more useful forms of energy, such as kinetic energy, into heat, which isn't generally useful. In many applications, the loss of useful energy is not particularly desirable, but there are some practical applications. Like:

- In the brakes of some trains. During braking, the brakes expose the metal wheels to a magnetic field which generates eddy currents in the wheels. The magnetic interaction between the applied field and the eddy currents slows the wheels down. The faster the wheels spin, the stronger is the effect, meaning that as the train slows the braking force is reduces, producing a smooth stopping motion.
- There are few galvanometers having a fixed core which are of nonmagnetic metallic material. When the coil oscillates, the eddy currents that generate in the core oppose the motion and bring the coil to rest.
- Induction furnace can be used to prepare alloys, by melting the metals. The eddy currents generated in the metals produce high temperature enough to melt it.



Applications of Electromagnetic Induction

1. Electromagnetic induction in AC generator
2. Electrical Transformers
3. Magnetic Flow Meter

FLEMMING RULE

Whenever a current carrying conductor comes under a magnetic field, there will be a force acting on the conductor. The direction of this force can be found using Fleming's Left Hand Rule (also known as 'Flemings left-hand rule for motors').

Similarly if a conductor is forcefully brought under a magnetic field, there will be an induced current in that conductor. The direction of this force can be found using Fleming's Right Hand

Rule.

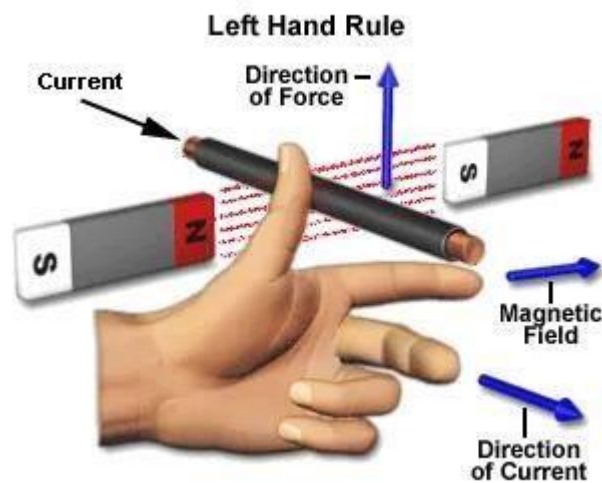
In both Fleming's left and right hand rules, there is a relation between the magnetic field, the current and force. This relation is directionally determined by **Fleming's Left Hand rule** and **Fleming's Right Hand rule** respectively.

These rules do not determine the magnitude but instead show the direction of any of the three parameters (magnetic field, current, force) when the direction of the other two parameters is known.

Fleming's Left-Hand rule is mainly applicable to electric motors and **Fleming's Right-Hand rule** is mainly applicable to electric generators.

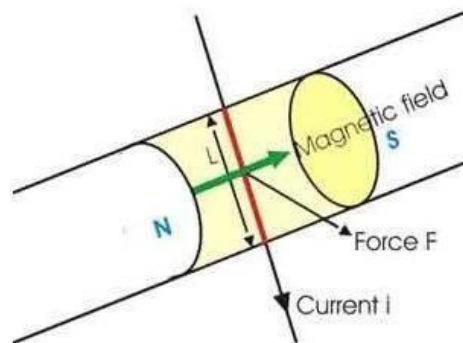
Fleming's Left Hand Rule

It is found that whenever a current carrying conductor is placed inside a magnetic field, a force acts on the conductor, in a direction perpendicular to both the directions of the current and the magnetic field.



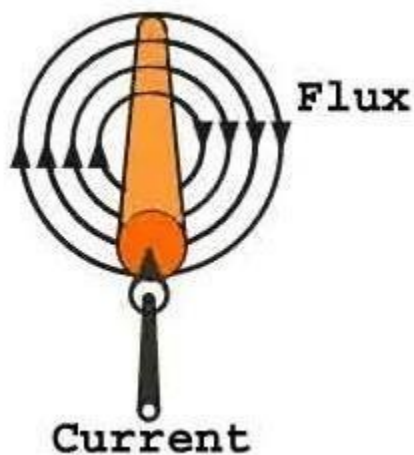
In the figure below, a portion of a conductor of length L is placed vertically in a uniform horizontal magnetic field of strength H , produced by two magnetic poles N and S. If the current I is flowing through this conductor, the magnitude of the force acting on the conductor is:

$$F = BiL$$



Hold out your left hand with the forefinger, second finger and thumb at the right angle to one another. If the forefinger represents the direction of the field and the second finger represents that of the current, then thumb gives the direction of the force.

While current flows through a conductor, one magnetic field is induced around it. The magnetic field can be imagined by considering numbers of closed magnetic lines of force around the conductor. The direction of magnetic lines of force can be determined by Maxwell's corkscrew rule or right-hand grip rule. As per these rules, the direction of the magnetic lines of force (or flux lines) is clockwise if the current is flowing away from the viewer, that is if the direction of current through the conductor is inward from the reference plane as shown in the figure.

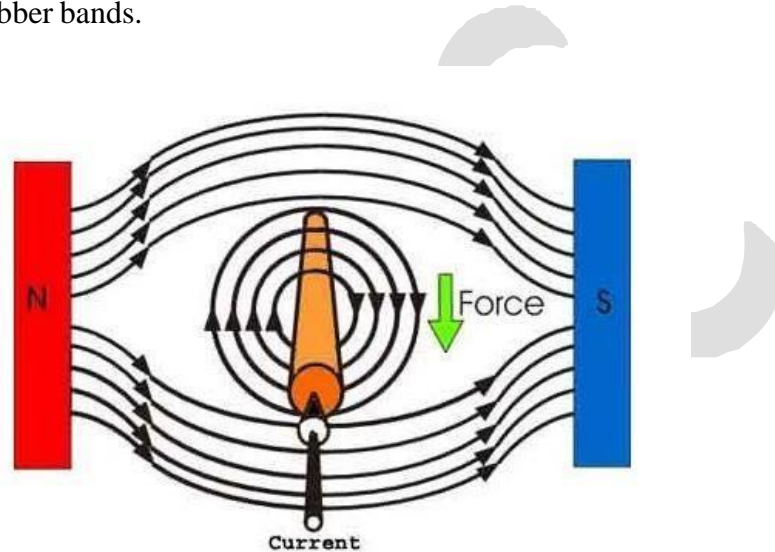


Now if a horizontal magnetic field is applied externally to the conductor, these two magnetic fields i.e. field around the conductor due to the current through it and the externally applied field will interact with each other. We observe in the picture that the magnetic lines of force of

external magnetic field are from N to S pole that is from left to right.

The magnetic lines of force of external magnetic field and magnetic lines of force due to the current in the conductor are in the same direction above the conductor, and they are in the opposite direction below the conductor. Hence there will be larger numbers of co-directional magnetic lines of force above the conductor than that of below the conductor.

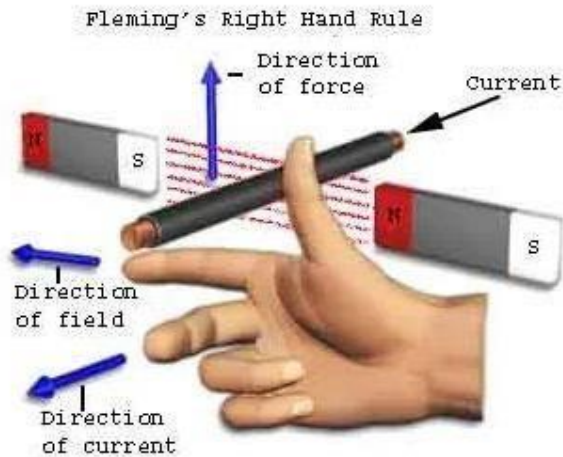
Consequently, there will be a larger concentration of magnetic lines of force in a small space above the conductor. As magnetic lines of force are no longer straight lines, they are under tension like stretched rubber bands.



As a result, there will be a force which will tend to move the conductor from the more concentrated magnetic field to less concentrated magnetic field, that is from the present position to downwards. Now if you observe the direction of the current, force and magnetic field in the above explanation, you will find that the directions are according to the Fleming left-hand rule.

Fleming Right Hand Rule

As per Faraday's law of electromagnetic induction, whenever a conductor moves inside a magnetic field, there will be an induced current in it. If this conductor gets forcefully moved inside the magnetic field, there will be a relation between the direction of applied force, magnetic field and the current. This relation among these three directions is determined by **Fleming's right-hand Rule**.



This rule states -Hold out the right hand with the first finger, second finger and thumb at the right angle to each other. If forefinger represents the direction of the line of force, the thumb points in the direction of motion or applied force, then second finger points in the direction of the induced current.

Who Invented The Left and Right Hand Thumb Rules?

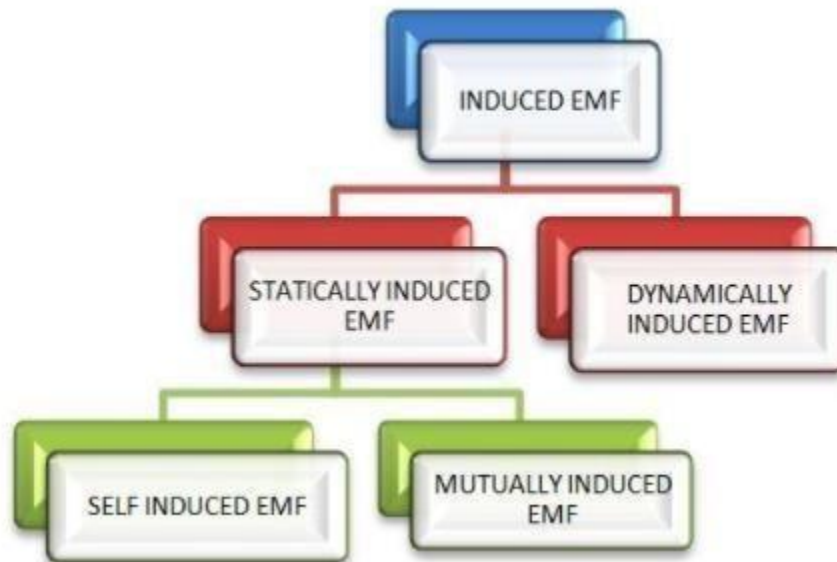
The left and right hand thumb rules were founded by John Ambrose Fleming in the late 19th century.

John discovered both of these rules and named them after himself. The rules are now well known as Fleming's left and right-hand rule.



STATICALLY INDUCED AND DYNAMICALLY INDUCED EMF

Induced e.m.f can be either dynamically induced emf or statically induced emf. In this first case, usually the field is stationary and conductors cut across it (as in d.c. generator). But in the second case, usually the conductor or the coil remains stationary and flux linked with it is changed by simply increasing or decreasing the current producing this flux (as in transformers).



Let the flux linking with the coil of turns N be changed by an amount $d\phi$ in short time dt .

EMF induced, $e =$ Rate of change of flux linkage

= Number of turns \times rate of change of flux

$$= N \frac{d\phi}{dt}$$

A minus sign is required to be placed before the right hand side quantity of above expression just to indicate the phenomenon explained by Lenz's law, therefore, expression for induced emf may be written as

$$e = -N \frac{d\phi}{dt} \text{ volts} \quad \dots\dots(1)$$

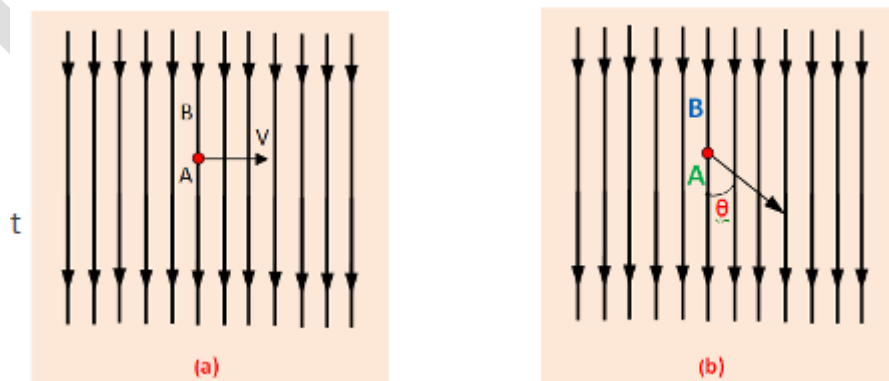
DYNAMICALLY INDUCED EMF

We have learnt that when the flux linking with the coil or circuit changes, an emf is induced in the coil or circuit.

EMF can be induced by changing the flux linking in two ways:

1. By increasing or decreasing the magnitude of the current producing the linking flux. In this case, there is no motion of the conductor or of coil relative to the field and, therefore, emf induced in this way is known as statically induced
2. By moving a conductor in a uniform magnetic field and emf produced in this way is known as *dynamically induced emf*

Consider a conductor of length l meters placed in a uniform magnetic field of density $B \frac{\text{Wb}}{\text{m}^2}$, as shown in Fig.



Let, l = Length of the conductor lying within the field. And it moves a distance dx in time dt ,
So, the area swept by the conductor is $l dx$. Hence, flux cut by the conductor = $l dx \times B$, Change

in Flux = $B.l.dx$ weber, Time= dt second

According to Faraday's laws, The e.m.f induced in the conductor . And this induced e.m.f is known as dynamically induced e.m.f.

The rate of change of flux linkages = $\frac{Bldx}{dt} = Bl \frac{dx}{dt} = Blv$ volt [Where, $\frac{dx}{dt}$ is velocity]

If the conductor (A) moves at an angle θ with the direction of flux which is shown in (b).

Then the induced e.m.f is, $e = Blv \sin \theta$ volts = $\vec{lv} \times \vec{B}$
(i.e as cross product vector \vec{v} and \vec{B}).

An example, the generator works on the production of dynamically induced e.m.f in the conductors.

The magnitude of emf induced, is proportional to the component of the velocity in a direction perpendicular to the direction of the magnetic field and induced emf is given by

$$e = Blv \sin \theta \text{ volts}$$

The direction of this induced emf is given by Fleming's right hand rule.

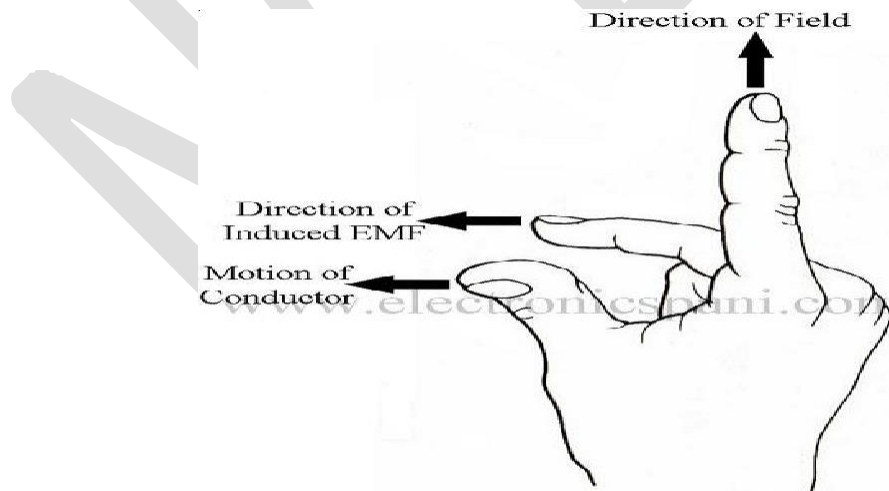


Figure : Fleming's Right Hand Rule

If the thumb, forefinger and middle finger of right hand are held mutually perpendicular to each other, forefinger pointing into the direction of the field and thumb in the direction of motion of

conductor then the middle finger will point in the direction of the induced emf as shown in Fig.

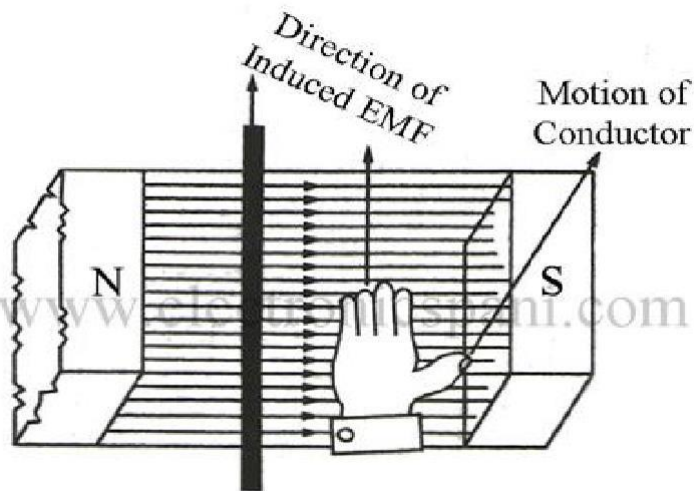


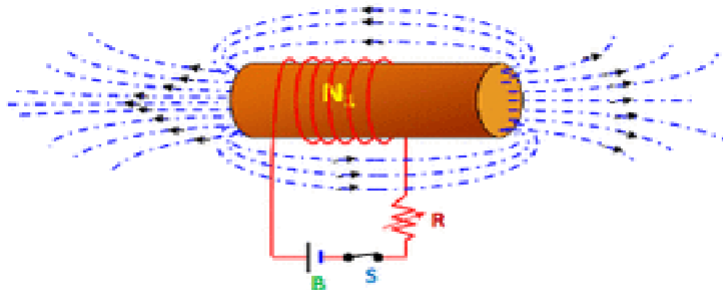
Figure Right Hand Flat Palm Rule

STATICALLY INDUCED E.M.F.:-

Statically induced e.m.f is two types which are –

- i) Mutually-induced e.m.f.
- ii) Self-induced e.m.f.

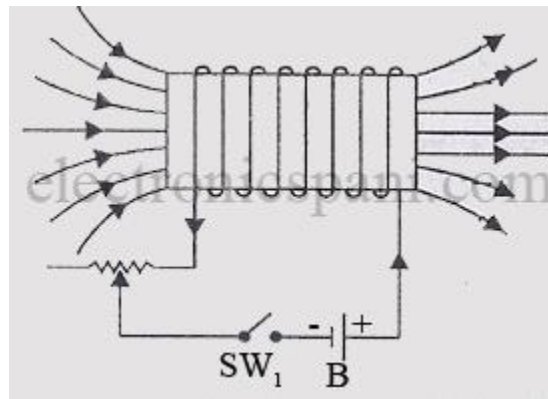
SELF-INDUCED E.M.F.:



Self-induced e.m.f is the e.m.f which is produced in the coil due to the change of its own flux

linked with it. If the current of the coil is changed, then the flux linked with its own turns will also change which will produce an e.m.f that is called self-induced e.m.f.

Since according to Lenz's law, an induced emf acts to oppose the change that produces it, a self-induced emf is always in such a direction as to oppose the change of current in the coil or circuit in which it is induced. This property of the coil or circuit due to which it opposes any change of the current in the coil or circuit, is known as self-inductance.



Consider a Solenoid of N turns, length l meters, area of X-section a square meters and of relative permeability μ_r . When the solenoid carries a current of i amperes, a magnetic field of

$$\frac{Ni}{l}$$

flux $\mu_0\mu_r a$ webers is set up around the solenoid and links with it.

If the current flowing through the solenoid is changed, the flux produced by it will change and, therefore, an emf will be induced.

$$e = -N \frac{d\phi}{dt} = -N \frac{d}{dt} \left[\frac{Ni}{l} \right]$$

self-induced emf,

$$= -N \frac{N}{l} \frac{di}{dt} \frac{1}{\mu_0\mu_r a}$$

$$= - \frac{N^2 \mu_r \mu_0 a}{l} \times \frac{di}{dt} \dots\dots(2)$$

The quantity $\frac{N^2 \mu_r \mu_0 a}{l}$ is a constant for any given coil or circuit and is called coefficient of self-

inductance. It is represented by symbol L and is measured in Henries.

Hence self-induced emf, $e = -L \frac{di}{dt}$

Where $L = \frac{N^2 \mu_r \mu_0 a}{l}$ henrys

Coefficient of Self Induction

The coefficient of self-induction (L) can be determined from any one of the following three relations.

First Method. In case the dimensions of the solenoid are given, the coefficient of self-induction may be determined from the relation

$$L = \frac{N^2 \mu_r \mu_0 a}{l} \text{ henrys} \quad \dots\dots(3)$$

Second Method. In case the magnitude of induced emf in a coil for a given rate of change of current in the coil is known, self-inductance of the coil may be determined from the following relation.

$$e = L \frac{di}{dt}$$

$$L = \frac{e}{\frac{di}{dt}} \quad \dots\dots(4)$$

Third Method. In case the number of turns of the coil and flux produced per ampere of current in the coil is known, the self-inductance of the coil may be determined from the following relation

$$L = \frac{N\phi}{i} \quad \dots\dots(5)$$

The above relation can be derived as follows:

Magnetic flux produced in a coil of N turns, length l meters, area of x-section a meters² and relative permeability μ_r when carrying a current of I amperes is given by

$$\phi = \frac{Ni}{\mu_r \mu_0 a}$$

and self-inductance of the coil

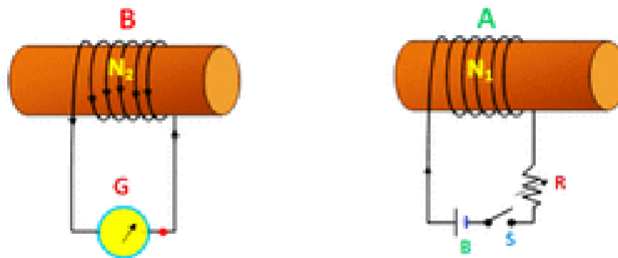
$$L = \frac{N^2 \mu_r \mu_0 a}{l} = \frac{N}{i} \frac{N \mu_r \mu_0 a}{l} i = \frac{N \phi}{i}$$

From the above relation, it is obvious that the self-inductance of a coil or circuit is equal to weber-turns per ampere in the coil or circuit.

In the above relation if $N\phi=1\text{Wb-turn}$ and $i = 1 \text{ A}$ then $L = 1 \text{ H}$.

Hence a coil is said to have a self-inductance of one henry if a current of 1 A, when flowing through it, produces flux linkage of 1 Wb-turn in it.

MUTUALLY INDUCED E.M.F:-



Mutually induced e.m.f occurs in between two coils. Let, A & B are two coils which are placed close to each other. If coil A is joined to a battery a switch and a variable resistance R and coil B is connected to a sensitive voltmeter G. When the switch S is closed, a current will flow through the coil A and produce a magnetic field in which partly links with the coil B. As current through A is changed, the flux linked with B is also changed. According to Faraday's law, induced e.m.f is produced in the coil B and This e.m.f know as mutually induce e.m.f.

In the above example, there is no movement of any conductor, the flux variation being brought about by variation in current strength Only. Such an e.m.f induced in one coil by influence of the

other coil is called mutually induced e.m.f.

Hence whenever the current in coil A changes, the flux linking with coil B changes and an emf, known as mutually induced emf is induced in coil B.

Consider coil A of turns N_1 wound on a core of length l meters, area of cross-section a square meters and relative permeability μ_r . When the current of i_1 amperes flows through it, a flux

$$\frac{N_1 i_1}{l}$$

of $\mu_0 \mu_r a$ is set up around the coil A.

Mutually induced emf, $e_m = -$ Rate of change of flux linkage of coil B

$= -N_2 \times$ rate of change of flux in coil A

$$= -N_2 \frac{d}{dt} \left[\frac{N_1 i_1}{l} \right]$$

$$= -N_2 \frac{N_1 a \mu_0 \mu_r}{l} \frac{di_1}{dt} \dots(6)$$

The quantity $\frac{N_1 N_2 a \mu_0 \mu_r}{l}$ is called the coefficient of mutual induction of coil B with respect to coil A. It is represented by symbol M and is measured in henrys.

Hence mutually induced emf,

$$e_m = -M \frac{di_1}{dt}$$

Where $M = \frac{N_1 N_2 a \mu_0 \mu_r}{l} \text{ henrys}$

Coefficient of Mutual Induction

Mutual inductance may be defined as the ability of one coil or circuit to induce an emf in a nearby coil by induction when the current flowing in the first coil is changed. The action is also reciprocal i.e. the change in current flowing through second coil will also induce an emf in the first coil. The ability of reciprocal induction is measured in terms of the coefficient of mutual induction M .

The coefficient of mutual induction (M) can be determined from any one of the following three relations.

First Method. In case the dimensions of the coils are given, the coefficient of mutual induction may be determined from the relation

$$L = \frac{N_1 N_2 a \mu_0 \mu_r}{l} \text{ henrys} \quad \dots(7)$$

Second Method. In case the magnitude of induced emf in the second coil for a given rate of change of current in the first coil is known, mutual inductance between the coil may be determined from the following relation

$$e_m = M \frac{di_1}{dt}$$

$$M = \frac{e_m}{\frac{di_1}{dt}} \quad \dots(8)$$

Third Method. In case the number of turns of the coil and flux linking with this coil per ampere of current in another coil is known, the mutual inductance of the coil may be determined from the following relation

$$M = N_2 \frac{\phi_2}{i_1} \text{ Henry} \quad \dots(9)$$

EXAMPLE PROBLEMS

1. A solenoid of 500 turns is wound on an iron core of relative permeability 800. The length and radius of the solenoid are 40 cm and 3 cm respectively. Calculate the average emf induced in the solenoid if the current in it changes from 0 to 3 A in 0.4 second.

Solution

$N = 500$ turns; $\mu_r = 800$;

$l = 40$ cm = 0.4 m; $r = 3$ cm = 0.03 m;

$di = 3 - 0 = 3$ A; $dt = 0.4$ s

Self inductance,

$$L = \mu n^2 Al \left(\because \mu = \mu_0 \mu_r; A = \pi r^2; n = \frac{N}{l} \right)$$

$$= \frac{\mu_0 \mu_r N^2 \pi r^2}{l}$$

$$= \frac{4 \times 3.14 \times 10^{-7} \times 800 \times 500^2 \times 3.14 \times (3 \times 10^{-2})^2}{0.4}$$

$$L = 1.77 \text{ H}$$

Magnitude of induced emf, $\epsilon = L \frac{di}{dt}$

$$= \frac{1.77 \times 3}{0.4}$$

$$\epsilon = 13.275 \text{ V}$$

2. The self-inductance of an air-core solenoid is 4.8 mH. If its core is replaced by iron core, then its self-inductance becomes 1.8 H. Find out the relative permeability of iron.

Solution

$$L_{air} = 4.8 \times 10^{-3} \text{ H}$$

$$L_{iron} = 1.8 \text{ H}$$

$$L_{air} = \mu_0 n^2 Al = 4.8 \times 10^{-3} \text{ H}$$

$$L_{iron} = \mu n^2 Al = \mu_r \mu_0 n^2 Al = 1.8 \text{ H}$$

$$\therefore \mu_r = \frac{L_{iron}}{L_{air}} = \frac{1.8}{4.8 \times 10^{-3}} = 375$$

3. The current flowing in the first coil changes from 2 A to 10 A in 0.4 sec. Find the mutual inductance between two coils if an emf of 60 mV is induced in the second coil. Also determine the induced emf in the second coil if the current in the first coil is changed from 4 A to 16 A in

0.03 sec. Consider only the magnitude of induced emf.

Solution

Case (i):

$$di_1 = 10 - 2 = 8 \text{ A}; dt = 0.4 \text{ s};$$

$$\epsilon_2 = 60 \times 10^{-3} \text{ V}$$

Case (ii):

$$di_1 = 16 - 4 = 12 \text{ A};$$

$$dt = 0.03 \text{ s}$$

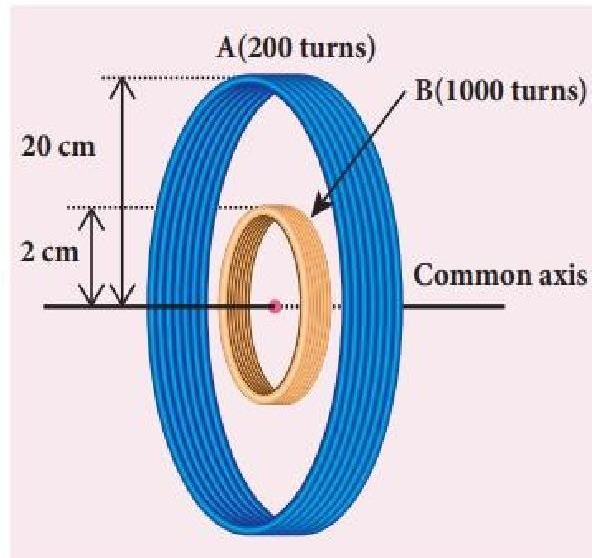
(i) Mutual inductance of the second coil with respect to the first coil

$$\begin{aligned} M_{21} &= \frac{\epsilon_2}{\frac{di_1}{dt}} \\ &= \frac{60 \times 10^{-3} \times 0.4}{8} \\ M_{21} &= 3 \times 10^{-3} \text{ H} \end{aligned}$$

(ii) Induced emf in the second coil due to the rate of change of current in the first coil is

$$\begin{aligned} \epsilon_2 &= M_{21} \frac{di_1}{dt} \\ &= \frac{3 \times 10^{-3} \times 12}{0.03} \\ \epsilon_2 &= 1.2 \text{ V} \end{aligned}$$

4. Consider two coplanar, co-axial circular coils A and B as shown in figure. The radius of coil A is 20 cm while that of coil B is 2 cm. The number of turns is 200 and 1000 for coils A and B respectively. Calculate the mutual inductance of coil B with respect to coil A. If the current in coil A changes from 2 A to 6 A in 0.04 sec, determine the induced emf in coil B and the rate of change of flux through the coil B at that instant.



Solution

$N_A = 200$ turns; $N_B = 1000$ turns;

$r_A = 20 \times 10^{-2}$ m; $r_B = 2 \times 10^{-2}$ m;

$dt = 0.04$ s; $di_A = 6 - 2 = 4$ A

Let i_A be the current flowing in coil A, then the magnetic field B_A at the centre of the circular coil A is

$$\begin{aligned}
 B_A &= \frac{\mu_0 N_A i_A}{2r_A} = \frac{4\pi \times 10^{-7} N_A i_A}{2r_A} \\
 &= \frac{10^{-7} \times 2 \times 3.14 \times 200}{20 \times 10^{-2}} \times i_A \\
 &= 6.28 \times 10^{-4} i_A \text{ Wbm}^{-2}
 \end{aligned}$$

The magnetic flux linkage of coil B is

$$\begin{aligned}
 N_B \Phi_B &= N_B B_A A_B \\
 &= 1000 \times 6.28 \times 10^{-4} \times i_A \times 3.14 \times (2 \times 10^{-2})^2 \\
 &= 7.89 \times 10^{-4} i_A \text{ Wb turns}
 \end{aligned}$$

The mutual inductance of the coil B with respect to coil A is

$$M_{BA} = \frac{N_B \Phi_B}{i_A} = 7.89 \times 10^{-4} \text{ H}$$

Induced emf in coil B is

$$\epsilon_B = -M_{BA} \frac{di_A}{dt}$$

Considering only the magnitude,

$$\epsilon_B = \frac{7.89 \times 10^{-4} \times (6-2)}{0.04}$$

$$\epsilon_B = \frac{7.89 \times 10^{-4} \times (4)}{4 \times 10^{-2}}$$

$$\epsilon_B = 7.89 \times 10^{-2} \text{ V}$$

The rate of change of magnetic flux of coil
 B is

$$\frac{d(N_B \Phi_B)}{dt} = \epsilon_B = 78.9 \text{ mWb s}^{-1}$$

The rate of change of magnetic flux of coil is

$$\frac{d(N_B \Phi_B)}{dt} = \epsilon_B = 78.9 \text{ mWb s}^{-1}$$

NCERC

ALTERNATIVE CURRENT FUNDAMENTALS

INTRODUCTION

An electrical circuit is a complete conductive path through which electrons flow from the source to the load and back to the source. The direction and magnitude of the electrons flow however depend on the kind of source. In **Electrical Engineering**, there are basically two types of voltage or current (Electrical Energy) source which defines the kind of circuit and they are; **Alternating Current (or voltage) and Direct Current**.

For the next couple of posts, we will be focusing on the Alternating current, and move through topics ranging from **what is Alternating current** to **AC wave forms** and so on.

AC Circuits

AC circuits as the name (Alternating Current) implies are simply circuits powered by an Alternating Source, either voltage or current. An **Alternating Current or Voltage**, is one in which the value of either the voltage or the current varies about a particular mean value and reverses direction periodically.

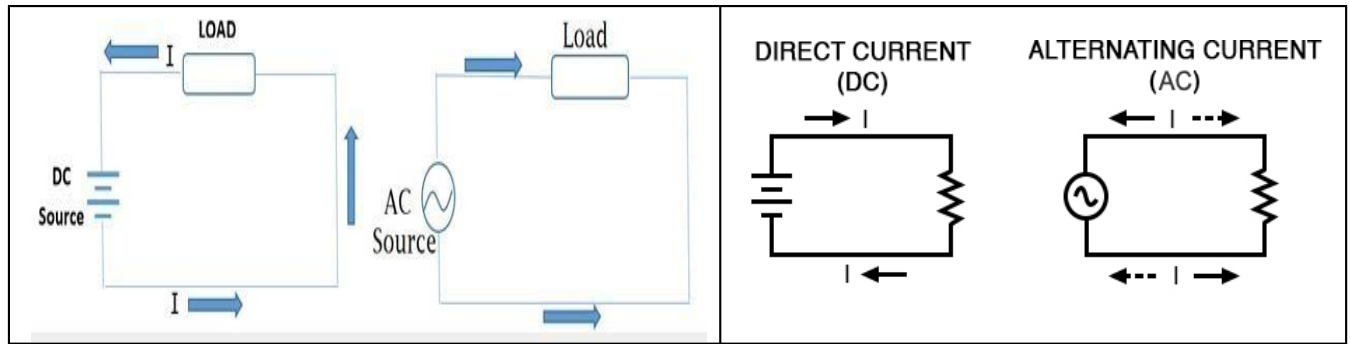
Most present day household and industrial Appliances and systems are powered using alternating current. All DC based plugged in appliances and rechargeable battery based devices technically run on Alternating current as they all use some form of DC power derived from AC for either charging of their batteries or powering of the system. Thus Alternating current is the form via which power is delivered at the mains.

The Alternating circuit came into being in the 1980s when Tesla decided to solve the long range incapability of the Thomas Edison's DC generators. He sought a way of transferring electricity at a high voltage and then employ the use of transformers to step it either up or down as may be needed for distribution and was thus able to minimize power loss across a great distance which was the main problem of Direct Current at the time.

Alternating Current VS Direct Current (AC vs DC)

AC and DC differ in several ways from generation to transmission, and distribution, but for the sake of simplicity, we will keep the comparison to their characteristics for this post.

The major difference between the AC and DC, which is also the cause of their different characteristics, is the direction of flow of electric energy. In DC, Electrons flow steadily in a single direction or forward, while in AC, electrons alternate their direction of flow in periodic intervals. This also leads to alternation in the voltage level as it switches along from positive to negative in line with the current.



Below is a comparison chart to highlight some of the **difference between AC and DC**. Other differences will be highlighted as we go more into exploring Alternating current Circuits.

Comparison Basis	AC	DC
Energy Transmission Capacity	Travels over long distance with minimal Energy loss	Large amount of energy is lost when sent over long distances
Generation Basics	Rotating a Magnet along a wire.	Steady Magnetism along a wire
Frequency	Usually 50Hz or 60Hz depending on Country	Frequency is Zero
Direction	reverses <u>direction</u> periodically when flowing through a circuit	It steady constant flow in one direction.
Current	Its Magnitude Vary with time	Constant Magnitude
Source	<u>All forms of AC Generators</u> and Mains	Cells, batteries, Conversion from AC
Passive Parameters	Impedance (RC, RLC, etc)	Resistance Only
Power Factor	Lies between 0&1	Always 1

Waveform	Sinusoidal, Trapezoidal, Triangular and Square	Straight line, sometimes Pulsating.
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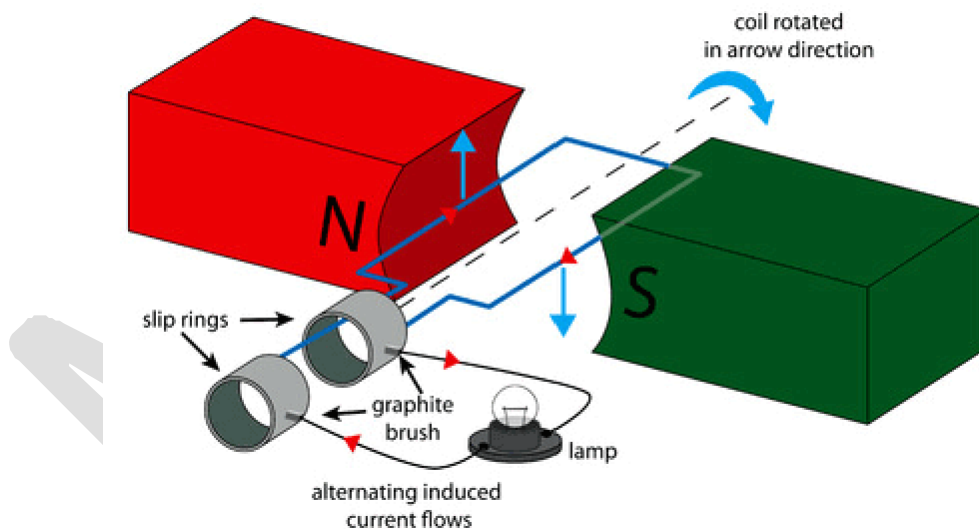
Basic AC Source generation (Single Coil AC Generator)

The **principle around AC generation** is simple. If a magnetic field or magnet is rotated along a stationary set of coils (wires) or the rotation of a coil around a stationary magnetic field, an Alternating current is generated using an AC generator(Alternator).

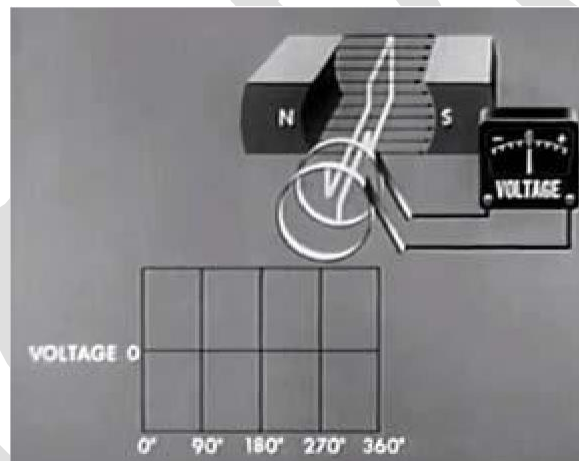
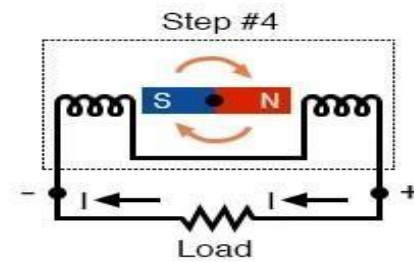
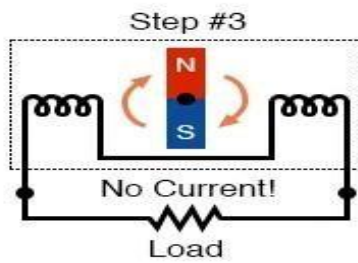
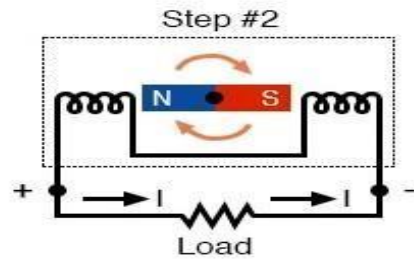
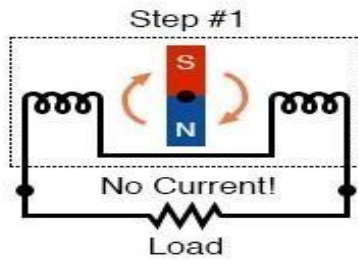
The simplest form of AC generator consists of a loop of wire that is mechanically rotated about an axis while positioned between the north and south poles of a magnet.

Consider the Image below.

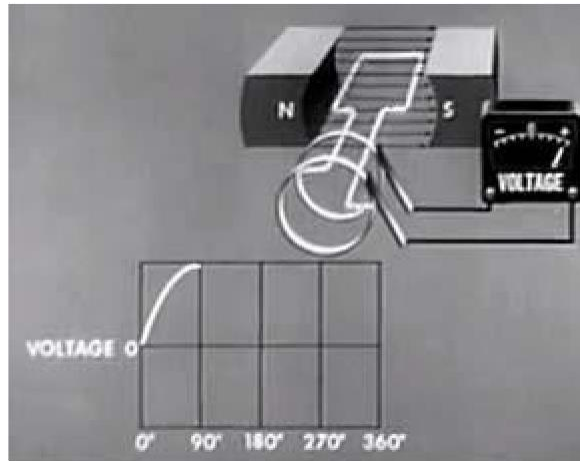
Simple a.c. Generator



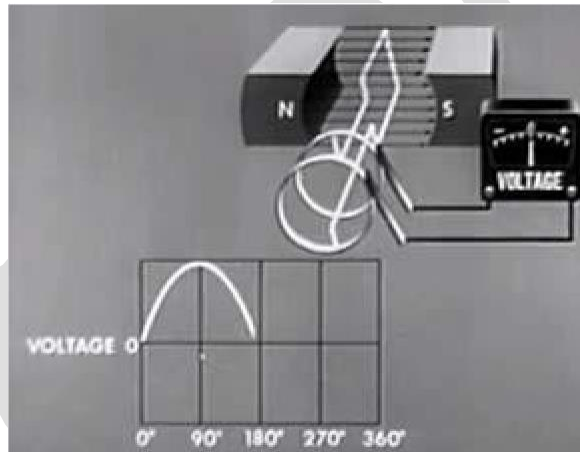
As the armature coil rotates within the magnetic field created by the north and south pole magnets, the magnetic flux through the coil changes, and charges are thus forced through the wire, giving rise to an effective voltage or induced voltage. The magnetic flux through the loop is as a result of the angle of the loop relative to the direction of the magnetic field. Consider the images below;



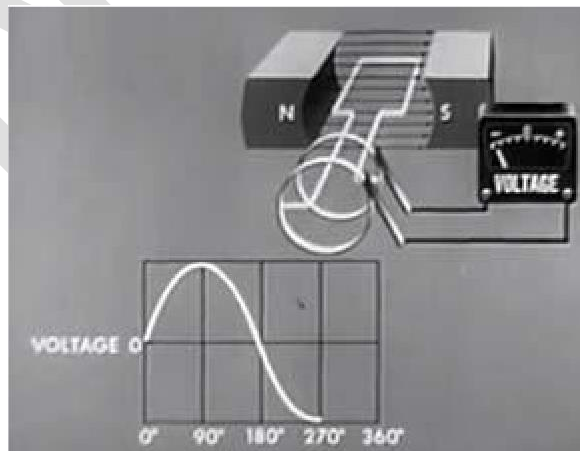
AC generator Armature at 0 degrees



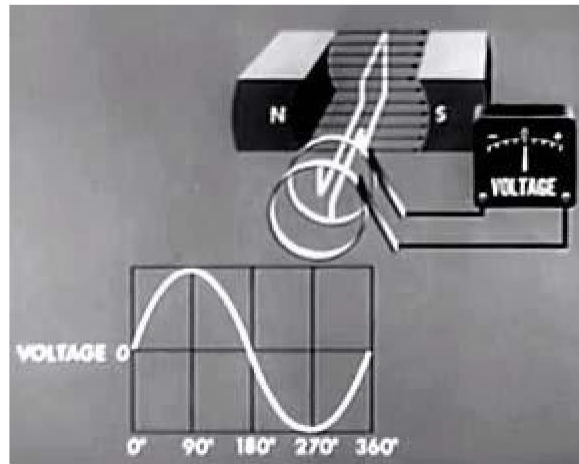
AC generator Armature at 90 degrees



AC generator Armature at 180 degrees



AC generator Armature at 270 degrees



AC generator Armature at 3

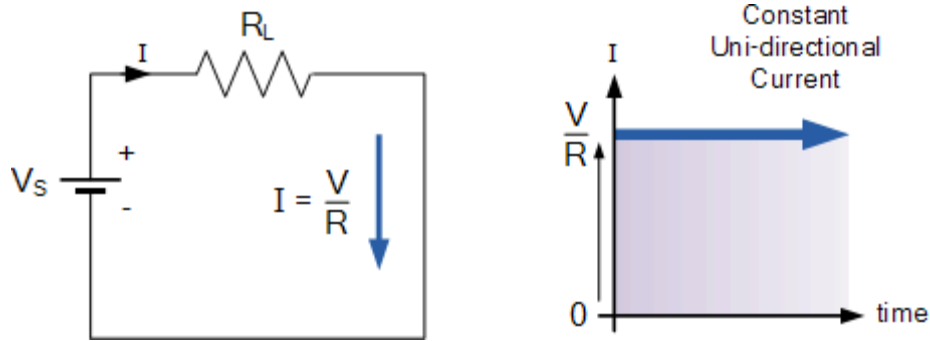
AC WAVEFORMS AND TERMINOLOGIES

Direct Current or **D.C.** as it is more commonly called, is a form of electrical current or voltage that flows around an electrical circuit in one direction only, making it a -Uni-directional supply.

Generally, both DC currents and voltages are produced by power supplies, batteries, dynamos and solar cells to name a few. A DC voltage or current has a fixed magnitude (amplitude) and a definite direction associated with it. For example, +12V represents 12 volts in the positive direction, or -5V represents 5 volts in the negative direction.

We also know that DC power supplies do not change their value with regards to time, they are a constant value flowing in a continuous steady state direction. In other words, DC maintains the same value for all times and a constant uni-directional DC supply never changes or becomes negative unless its connections are physically reversed. An example of a simple DC or direct current circuit is shown below.

DC Circuit and Waveform



An alternating function or **AC Waveform** on the other hand is defined as one that varies in both magnitude and direction in more or less an even manner with respect to time making it a **Bi-directional** waveform. An AC function can represent either a power source or a signal source with the shape of an *AC waveform* generally following that of a mathematical sinusoid being defined as: $A(t) = A_{\max} \cdot \sin(2\pi ft)$.

The term AC or to give it its full description of **Alternating Current**, generally refers to a time-varying waveform with the most common of all being called a **Sinusoid** better known as a **Sinusoidal Waveform**. Sinusoidal waveforms are more generally called by their short description as **Sine Waves**. Sine waves are by far one of the most important types of AC waveform used in electrical engineering.

The shape obtained by plotting the instantaneous ordinate values of either voltage or current against time is called an **AC Waveform**. An AC waveform is constantly changing its polarity every half cycle alternating between a positive maximum value and a negative maximum value respectively with regards to time with a common example of this being the domestic mains voltage supply we use in our homes.

This means then that the *AC Waveform* is a **time-dependent** signal with the most common type of time-dependant signal being that of the **Periodic Waveform**. The periodic or AC waveform is the resulting product of a rotating electrical generator. Generally, the shape of any periodic waveform can be generated using a fundamental frequency and superimposing it with harmonic signals of varying frequencies and amplitudes but that's for another tutorial.

Alternating voltages and currents can not be stored in batteries or cells like direct current (DC) can, it is much easier and cheaper to generate these quantities using alternators or waveform generators when they are needed. The type and shape of an AC waveform depends upon the generator or device producing them, but all AC waveforms consist of a zero voltage line that divides the waveform into two symmetrical halves. The main characteristics of an **AC Waveform** are defined as:

AC Waveform Characteristics

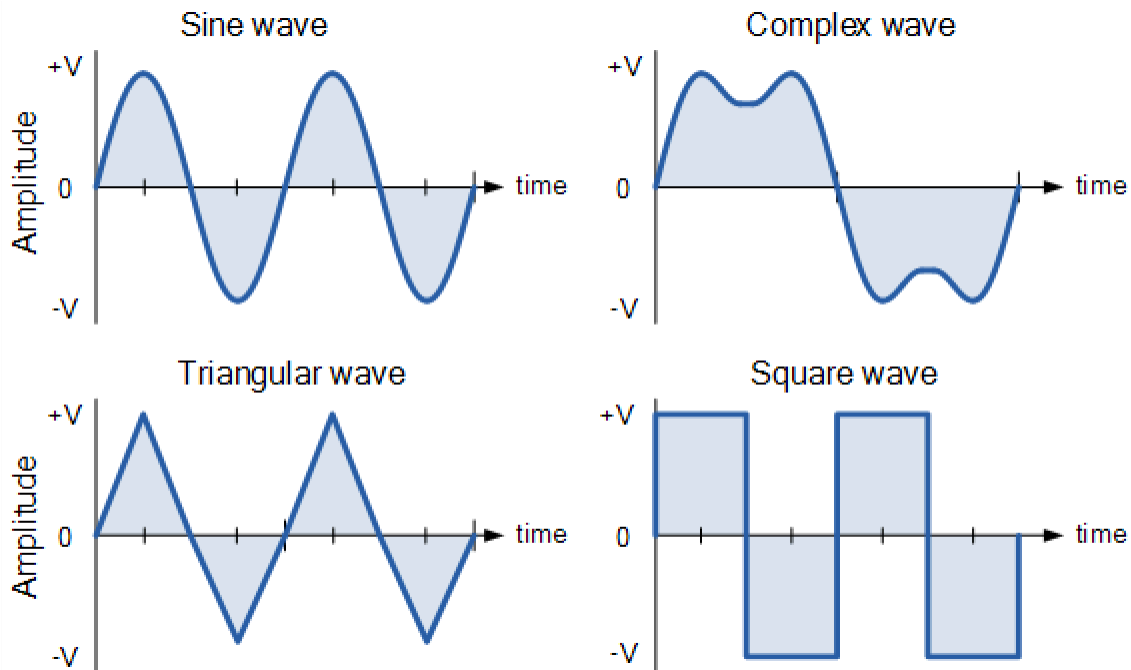
- The Period, (T) is the length of time in seconds that the waveform takes to repeat itself from start to finish. This can also be called the *Periodic Time* of the waveform for sine waves, or the *Pulse Width* for square waves.
- The Frequency, (f) is the number of times the waveform repeats itself within a one second time period. Frequency is the reciprocal of the time period, ($f = 1/T$) with the unit of frequency being the *Hertz*, (Hz).
- The Amplitude (A) is the magnitude or intensity of the signal waveform measured in volts or amps.

In our tutorial about **Waveforms** ,we looked at different types of waveforms and said that **-Waveforms** are basically a visual representation of the variation of a voltage or current plotted to a base of time. Generally, for AC waveforms this horizontal base line represents a zero condition of either voltage or current. Any part of an AC type waveform which lies above the horizontal zero axis represents a voltage or current flowing in one direction.

Likewise, any part of the waveform which lies below the horizontal zero axis represents a voltage or current flowing in the opposite direction to the first. Generally for sinusoidal AC waveforms the shape of the waveform above the zero axis is the same as the shape below it. However, for most non-power AC signals including audio waveforms this is not always the case.

The most common periodic signal waveforms that are used in Electrical and Electronic Engineering are the *Sinusoidal Waveforms*. However, an alternating AC waveform may not always take the shape of a smooth shape based around the trigonometric sine or cosine function. AC waveforms can also take the shape of either *Complex Waves*, *Square Waves* or *Triangular Waves* and these are shown below.

Types of Periodic Waveform



The time taken for an **AC Waveform** to complete one full pattern from its positive half to its negative half and back to its zero baseline again is called a **Cycle** and one complete cycle contains both a positive half-cycle and a negative half-cycle. The time taken by the waveform to complete one full cycle is called the **Periodic Time** of the waveform, and is given the symbol -Tl.

The number of complete cycles that are produced within one second (cycles/second) is called the **Frequency**, symbol f of the alternating waveform. Frequency is measured in **Hertz**, (Hz) named after the German physicist Heinrich Hertz.

Then we can see that a relationship exists between cycles (oscillations), periodic time and frequency (cycles per second), so if there are f number of cycles in one second, each individual cycle must take $1/f$ seconds to complete.

Relationship Between Frequency and Periodic Time

$$\text{Frequency, } (f) = \frac{1}{\text{Periodic Time}} = \frac{1}{T} \text{ Hertz}$$

or

$$\text{Periodic Time, } (T) = \frac{1}{\text{Frequency}} = \frac{1}{f} \text{ seconds}$$

AC Waveform Example No1

1. What will be the periodic time of a 50Hz waveform and 2. what is the frequency of an AC waveform that has a periodic time of 10mS.

1).

$$\text{Periodic Time, (T)} = \frac{1}{f} = \frac{1}{50} = 0.02\text{secs or } 20\text{ms}$$

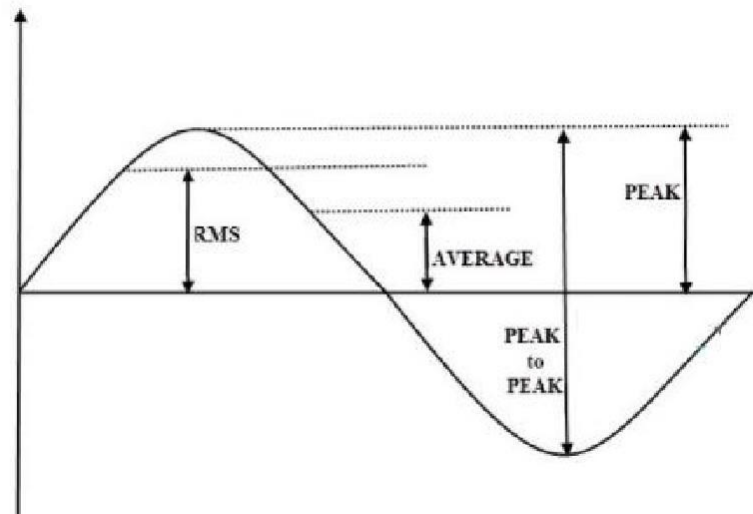
2).

$$\text{Frequency, (f)} = \frac{1}{T} = \frac{1}{10 \times 10^{-3}} = 100\text{Hz}$$

Frequency used to be expressed in -cycles per second abbreviated to -cps, but today it is more commonly specified in units called -Hertz. For a domestic mains supply the frequency will be either 50Hz or 60Hz depending upon the country and is fixed by the speed of rotation of the generator. But one hertz is a very small unit so prefixes are used that denote the order of magnitude of the waveform at higher frequencies such as **kHz**, **MHz** and even **GHz**.

Definition of Frequency Prefixes

Prefix	Definition	Written as	Periodic Time
Kilo	Thousand	kHz	1ms
Mega	Million	MHz	1us
Giga	Billion	GHz	1ns
Terra	Trillion	THz	1ps



Amplitude of an AC Waveform

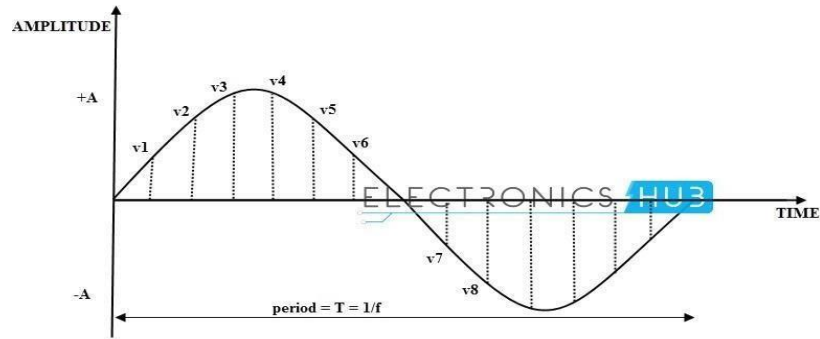
As well as knowing either the periodic time or the frequency of the alternating quantity, another important parameter of the AC waveform is **Amplitude**, better known as its Maximum or Peak value represented by the terms, V_{max} for voltage or I_{max} for current.

The peak value is the greatest value of either voltage or current that the waveform reaches during each half cycle measured from the zero baseline. Unlike a DC voltage or current which has a steady state that can be measured or calculated using **Ohm's Law**, an alternating quantity is constantly changing its value over time.

For pure sinusoidal waveforms this peak value will always be the same for both half cycles ($+V_m = -V_m$) but for non-sinusoidal or complex waveforms the maximum peak value can be very different for each half cycle. Sometimes, alternating waveforms are given a *peak-to-peak*, V_{p-p} value and this is simply the distance or the sum in voltage between the maximum peak value, $+V_{max}$ and the minimum peak value, $-V_{max}$ during one complete cycle.

Instantaneous Voltage

Instantaneous voltage is the voltage between two points at a particular moment in time. The voltage of a waveform at a given instant in time is called -Instantaneous voltage.



In the above diagram $v_1, v_2, v_3, v_4, v_5, v_6, \dots$ are the instantaneous voltages of the sine wave.

To find the instantaneous voltage value of the sine wave, we depend on Maximum voltage of the sine wave.

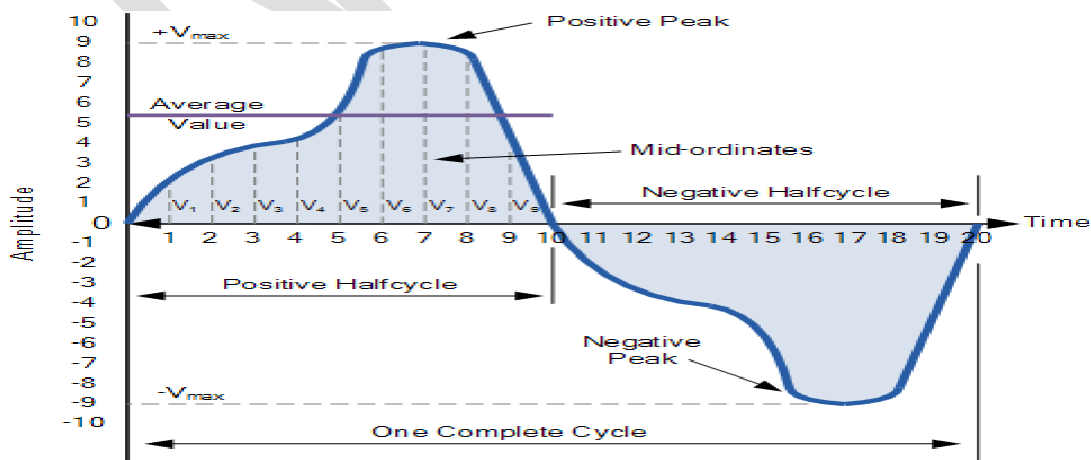
Instantaneous voltage = Maximum voltage $\times \sin \theta$

$$V_{\text{inst}} = V_{\text{max}} \times \sin \theta$$

The Average Value of an AC Waveform

The average or mean value of a continuous DC voltage will always be equal to its maximum peak value as a DC voltage is constant. This average value will only change if the duty cycle of the DC voltage changes. In a pure sine wave if the average value is calculated over the full cycle, the average value would be equal to zero as the positive and negative halves will cancel each other out. So the average or mean value of an AC waveform is calculated or measured over a half cycle only and this is shown below.

Average Value of a Non-sinusoidal Waveform



To find the average value of the waveform we need to calculate the area underneath the waveform using the mid-ordinate rule, trapezoidal rule or the Simpson's rule found commonly in mathematics. The approximate area under any irregular waveform can easily be found by simply using the mid-ordinate rule.

The zero axis base line is divided up into any number of equal parts and in our simple example above this value was nine, (V_1 to V_9). The more ordinate lines that are drawn the more accurate will be the final average or mean value. The average value will be the addition of all the instantaneous values added together and then divided by the total number. This is given as.

Average Value of an AC Waveform

$$V_{\text{average}} = \frac{V_1 + V_2 + V_3 + V_4 + \dots + V_n}{n}$$

Where: n equals the actual number of mid-ordinates used.

For a pure sinusoidal waveform this average or mean value will always be equal to $0.637 * V_{\text{max}}$ and this relationship also holds true for average values of current.

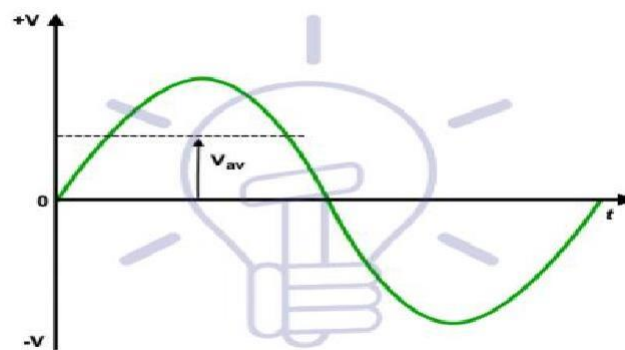


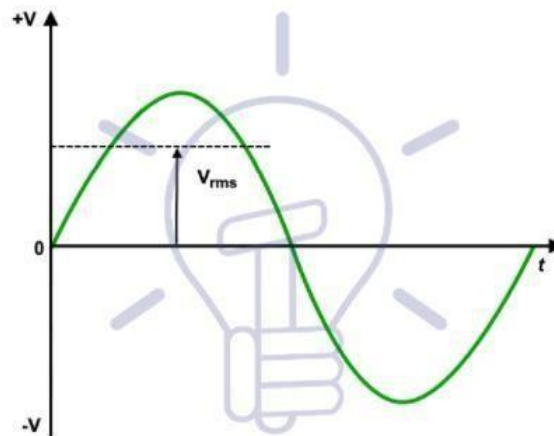
Fig - Average Value of Voltage

The RMS Value of an AC Waveform

The average value of an AC waveform that we calculated above as being: $0.637 * V_{\text{max}}$ is NOT the same value we would use for a DC supply. This is because unlike a DC supply which is constant and of a fixed value, an AC waveform is constantly changing over time and has no fixed value. Thus the equivalent value for an alternating current system that provides the same amount of electrical power to a load as a DC equivalent circuit is called the -effective value.

The effective value of a sine wave produces the same $I^2 * R$ heating effect in a load as we would expect to see if the same load was fed by a constant DC supply. The effective value of a sine wave is more commonly known as the **Root Mean Squared** or simply **RMS** value as it is

calculated as the square root of the mean (average) of the square of the voltage or current.



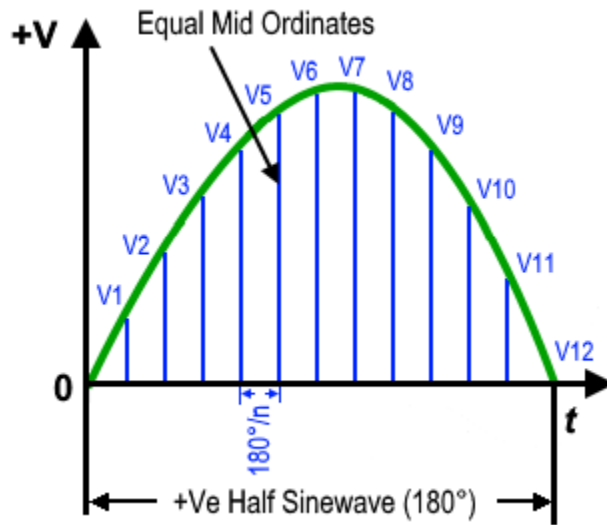
That is V_{rms} or I_{rms} is given as the square root of the average of the sum of all the squared mid-ordinate values of the sine wave. The RMS value for any AC waveform can be found from the following modified average value formula as shown.

RMS Value of an AC Waveform

$$V_{RMS} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + V_4^2 + \dots + V_n^2}{n}}$$

Where: n equals the number of mid-ordinates.

For a pure sinusoidal waveform this effective or R.M.S. value will always be equal to: $1/\sqrt{2} * V_{max}$ which is equal to $0.707 * V_{max}$ and this relationship holds true for RMS values of current. The RMS value for a sinusoidal waveform is always greater than the average value except for a rectangular waveform. In this case the heating effect remains constant so the average and the RMS values will be the same.



One final comment about R.M.S. values. Most multimeters, either digital or analogue unless otherwise stated only measure the R.M.S. values of voltage and current and not the average. Therefore when using a multimeter on a direct current system the reading will be equal to $I = V/R$ and for an alternating current system the reading will be equal to $I_{rms} = V_{rms}/R$.

Also, except for average power calculations, when calculating RMS or peak voltages, only use V_{RMS} to find I_{RMS} values, or peak voltage, V_p to find peak current, I_p values. Do not mix them together as Average, RMS or Peak values of a sine wave are completely different and your results will definitely be incorrect.

RMS Voltage Value Formulas for Different Wave forms

In the below table, the RMS Voltage Value formulas are shown for different kind of sinusoidal wave forms.

Waveform Type	Formula for RMS Value (V_{RMS})
Sine Wave	$V_{PK} / \sqrt{2}$
Half wave rectified sine wave	$V_{PK} / \sqrt{2}$
Full wave rectified sine wave	$V_{PK} / \sqrt{2}$
Square wave	V_{PK}

Triangle waveform	$V_{PK} / \sqrt{3}$
Sawtooth waveform	$V_{PK} / \sqrt{3}$

Form Factor and Crest Factor

Although little used these days, both **Form Factor** and **Crest Factor** can be used to give information about the actual shape of the AC waveform. Form Factor is the ratio between the average value and the RMS value and is given as.

$$\text{Form Factor} = \frac{\text{R.M.S value}}{\text{Average value}} = \frac{0.707 \times V_{\max}}{0.637 \times V_{\max}}$$

For a pure sinusoidal waveform the Form Factor will always be equal to 1.11. Crest Factor is the ratio between the R.M.S. value and the Peak value of the waveform and is given as.

$$\text{Crest Factor} = \frac{\text{Peak value}}{\text{R.M.S. value}} = \frac{V_{\max}}{0.707 \times V_{\max}}$$

For a pure sinusoidal waveform the Crest Factor will always be equal to 1.414.

AC Waveform Example No2

A sinusoidal alternating current of 6 amps is flowing through a resistance of 40Ω . Calculate the average voltage and the peak voltage of the supply.

The R.M.S. Voltage value is calculated as:

$$V_{\text{RMS}} = I \times R = 6 \times 40 = 240\text{V}$$

The Average Voltage value is calculated as:

$$\text{Form Factor} = \frac{V_{\text{RMS}}}{V_{\text{average}}}$$

$$\therefore V_{\text{average}} = \frac{V_{\text{RMS}}}{\text{Form Factor}} = \frac{240}{1.11} = 216.2 \text{ volts}$$

The Peak Voltage value is calculated as:

$$\text{Peak Voltage} = \text{R.M.S.} \times 1.414$$

$$\therefore 240 \times 1.414 = 339.4 \text{ volts}$$

The use and calculation of Average, R.M.S, Form factor and Crest Factor can also be use with any type of periodic waveform including Triangular, Square, Sawtoothed or any other irregular or complex voltage/current waveform shape. Conversion between the various sinusoidal values can sometimes be confusing so the following table gives a convenient way of converting one sine wave value to another.

Sinusoidal Waveform Conversion Table

Convert From	Multiply By	Or By	To Get Value
Peak	2	$(\sqrt{2})^2$	Peak-to-Peak
Peak-to-Peak	0.5	1/2	Peak
Peak	0.707	$1/(\sqrt{2})$	RMS
Peak	0.637	$2/\pi$	Average
Average	1.570	$\pi/2$	Peak
Average	1.111	$\pi/(2\sqrt{2})$	RMS

RMS	1.414	$\sqrt{2}$	Peak
RMS	0.901	$(2\sqrt{2})/\pi$	Average

Example Problem 3

If a sine wave is defined as $V_m = 150 \sin(220t)$, then find its RMS velocity and frequency and instantaneous velocity of the waveform after a 5 ms of time.

Solution:

The general equation for the sine wave is $V_t = V_m \sin(\omega t)$

Comparing this to the given equation $V_m = 150 \sin(220t)$,

The peak voltage of the maximum voltage is 150 volts and

Angular frequency is 220 rad / sec.

The RMS velocity of the wave form is given as

$V_{rms} = 0.707 \times \text{max amplitude or peak value.}$

$= 0.707 \times 150 = 106.05 \text{ volts}$

The angle of a sine wave is a function of its frequency, as we know the sine wave's angular velocity, so we can find out the frequency of the waveform. By using the relation between ω and f

Angular velocity (ω) =

Frequency (f) = $\omega / 2\pi$

For the given sine wave form $\omega = 220$,

Frequency = $220 / 2\pi$

$= 220 / (2 \times 3.1416)$

$= 220 / 6.2832$

$$= 35.0140 \text{ Hz}$$

The instantaneous value is given by after a time of 5 ms can be calculated by using the below formula.

$$V_i = 150 \sin (220 \times 5 \text{ ms})$$

$$= 150 \sin (1.1)$$

$$= 150 \times 0.019$$

$$= 133.68 \text{ volts}$$

Phase of the angle at time $t = 5 \text{ ms}$ are calculated in radians. We can convert the radian values into degree values very simply. The formula for conversion of radians to degrees is

$$\text{Degrees} = (1800 / \pi) \times \text{radians}$$

Converting 1.1 radians into degrees,

$$= (1800 / \pi) \times 1.1$$

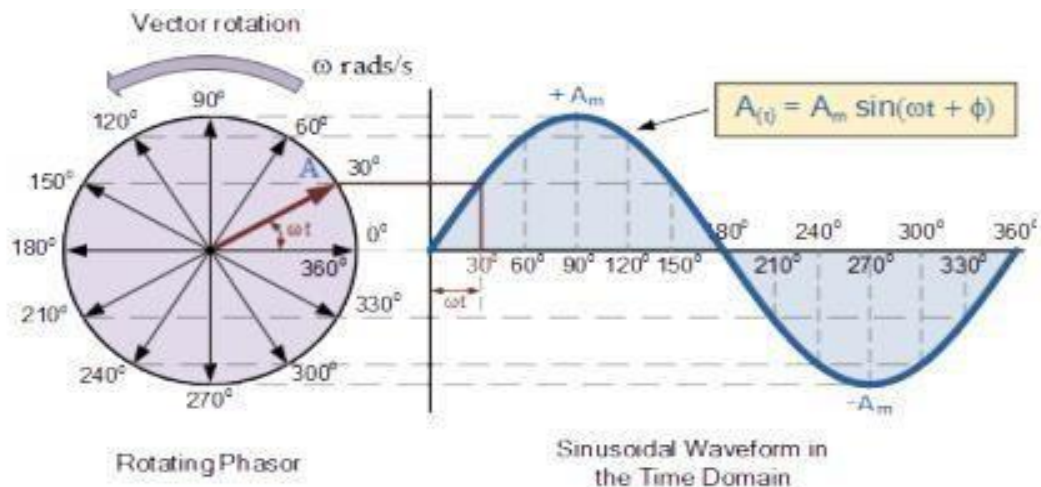
$$= 63.02 \text{ degrees}$$

MODULE III

AC CIRCUITS

SINUSOIDAL REPRESENTATION OF AC

The sinusoidal waveform $A(t) = A_m \sin(\omega t + \phi)$ could be expressed as a vector, rotating anti-clockwise with an angular frequency ω



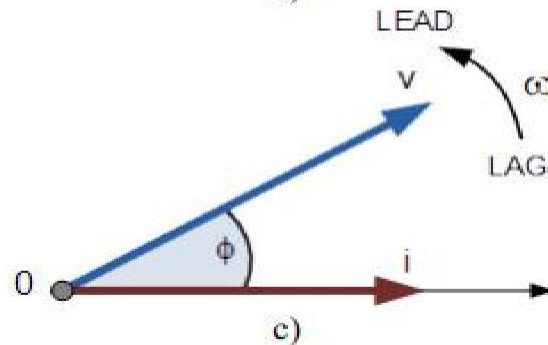
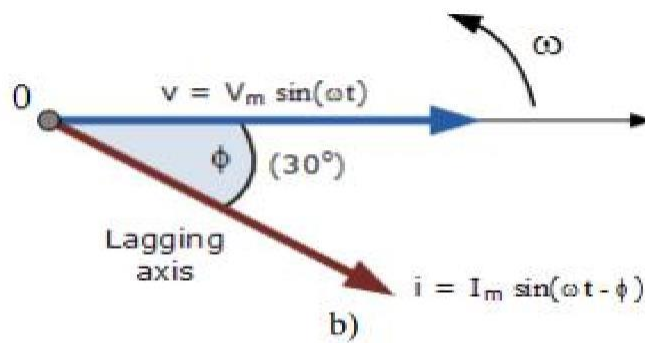
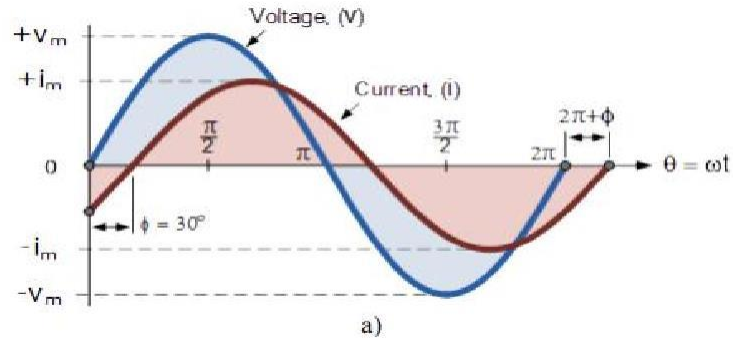
As can be seen when the time is $t=0$ the vector is rotated at $0^\circ, 180^\circ$ and 360° . Similarly when $A(t)$ has a maximum ($+A_m$) the vector is rotated at 90° and when $A(t)$ has a minimum ($-A_m$) – the vector is rotated at 270° .

Consider the current and voltage of a branch are:

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t - 30^\circ)$$

The current lags the voltage by $\phi = 30^\circ$ (fig.). Then the phasor diagram of the two vectors for $t=0$ is presented in fig. 5.6b. In time the two vectors rotate together with angular frequency ω however the current vector will continue to lag the voltage by 30°

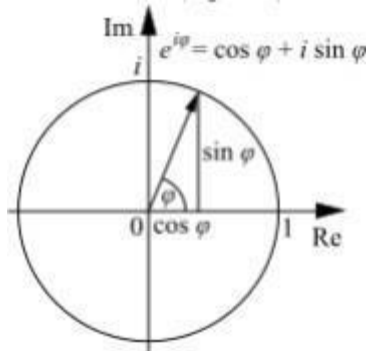


The sinusoidal waveform $A(t) = A_m \cdot \sin(\omega t + \phi)$ could be expressed in phasor form as:

$$\hat{A} \cdot m = A_m \cdot e^{j\phi} = A_m \cos(\phi) + j A_m \sin(\phi)$$

where $A \cdot m$ is also called complex amplitude.

The above equation is called the Euler's formula

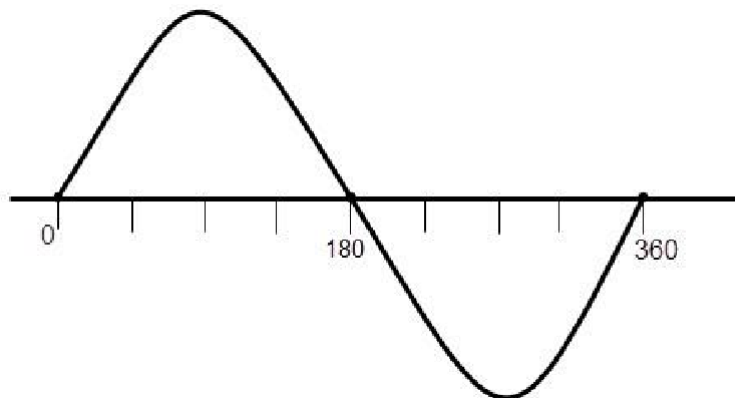


PHASE:

The position of a wave particle of a periodic waveform is known as “Phase” of a waveform. The complete phase of a full cycle of a waveform is 360° .

When two or more waves of the same frequency are interfering in a medium or made to travel in the same path, then the phase of waves play an important role to produce the desired output without any noise occurring in it.

Phase can also be defined as -The relative displacement of two waves with respect to each other.



Phase can be expressed in radians and degrees also. One radian = 57.3 degrees.

PHASE DIFFERENCE

The phase difference of a sine wave can be defined as -The time interval by which a wave leads by or lags by another wave and the phase difference is not a property of only one wave, it's the

relative property to two or more waves. This is also called as -Phase angle| or -Phase offset|.

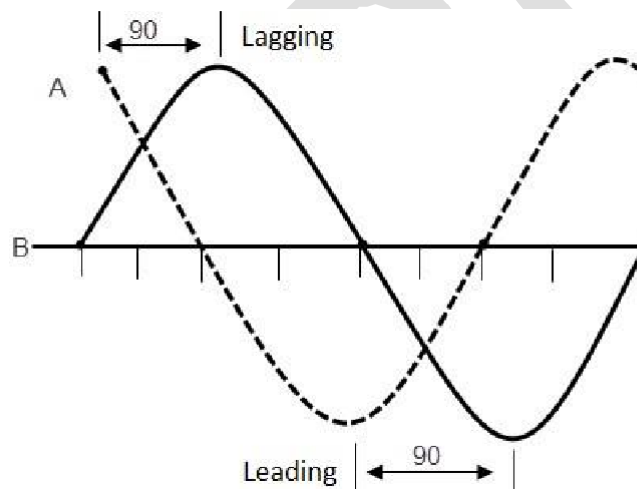
The phase difference represented by the Greek letter Phi (Φ). The complete phase of a waveform can be defined as 2π radians or 360 degrees.

Leading phase means, a waveform is ahead of another wave with the same frequency and Lagging phase means, a waveform is behind another wave with the same frequency.

Phase quadrature: When the phase difference between two waves is 90° (it may be $= + 90^\circ$ or $- 90^\circ$), then the waves are said to be in 'Phase quadrature'.

Phase opposition: When the phase difference between two waves is 180° (it may be $= + 180^\circ$ or $- 180^\circ$), then the waves are said to be in 'Phase opposition'.

To understand this concept clearly, observe the figure below.



The time interval and phase of a waveform are inversely proportional to each other. It means

$$t \text{ deg} = 1 / (360 f) \text{ (Degrees)}$$

$$t \text{ rad} = 1 / (6.28 f) \text{ (Radians)}$$

Where f is the frequency of the waveform and t is the time period.

For example, if two sine waves have the same frequency and have a phase shift of $\pi/2$ radians, then the phases of the waves can be defined as $(n\pi + 1)$ and $n\pi$ radians.

The phase shift of the waveforms can be represented in time period (T) also. For example $+ 6\text{ms}$ and $- 7\text{ms}$ etc.

Phase Difference Equation

The phase difference of sine waveforms can be expressed by below given equation, using

maximum voltage or amplitude of the wave forms,

$$A(t) = A_{\max} \times \sin(\omega t \pm \Phi)$$

Where

A_{\max} is the amplitude of the measured sine wave

ω is the angular velocity (radians / Sec)

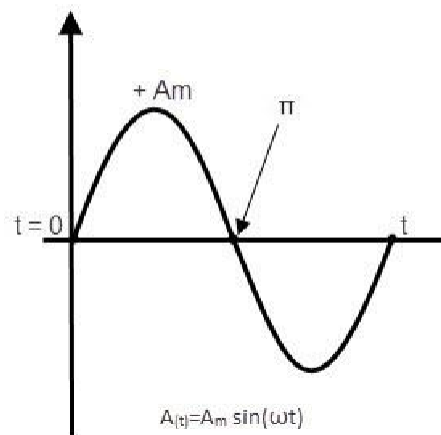
Φ is the phase angle. (Radians or degrees)

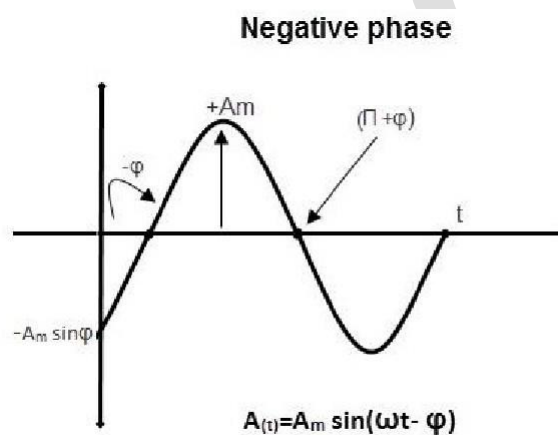
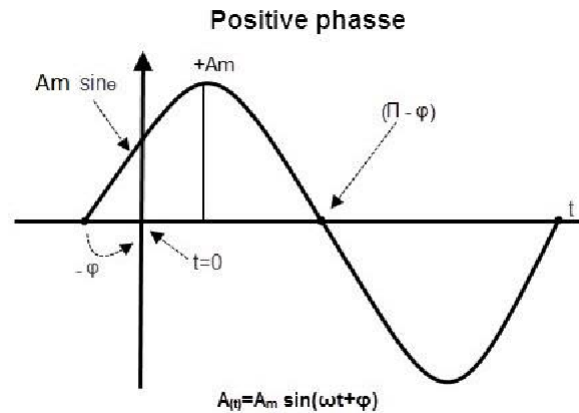
If $\Phi < 0$, then the phase angle of the wave is said to be in negative phase. Similarly, if $\Phi > 0$, then the phase angle of the wave is said to be in a positive phase.

Phase Relationship of a Sinusoidal Waveform

Every alternating waveform will have its current, voltage and frequency. If the voltage and angular velocities of the two waveforms are same, then their phase is also same at any instant of time.

In-phase



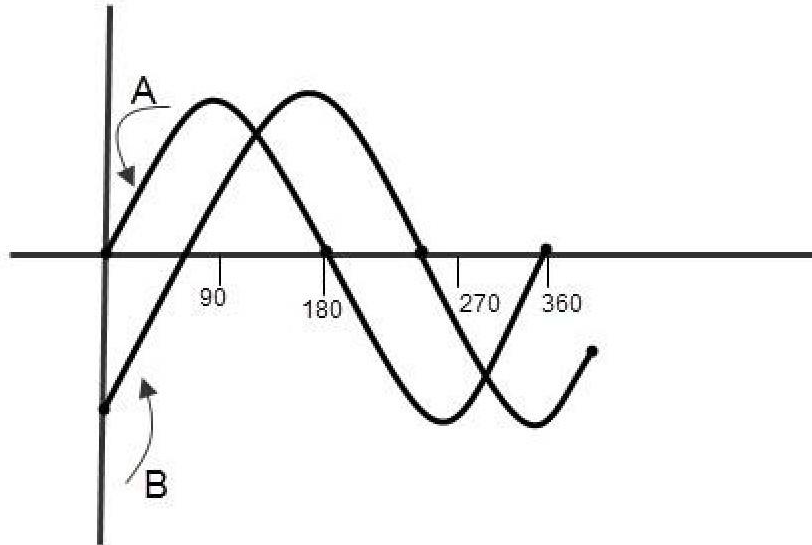


In the above picture, we can see three waves which are starting at the origin of the axis, leading at origin and lagging at the origin of the coordinate axis respectively.

PHASE DIFFERENCE OF WAVEFORMS

Out of Phase

When alternating waveforms have the same frequency but different phase, they are said to be -Out of phase. The phase difference is not zero for out of phased waves. Observe the below figure which describes the out of phase concept of two sine waves. For in phase waveforms, the retardation is fractions of numbers of wavelengths like $1/2, 2/3, 3/5 \dots$ etc.



In the above picture, the wave 'B' leads by 90° ($\Phi = 90^{\circ}$) to wave 'A'. So we can say that the two waves are out – of – phase.

For out of phased waves, there are two conditions. They are

1. Leading phase
2. Lagging phase

Leading Phase

When two waveforms of the same frequency are travelling along the same axis and one waveform is ahead of another, then it is called 'Leading phase wave'.

The current and voltage equations for leading phased waveforms are

$$\text{Voltage (Vt)} = V_m \sin \omega t$$

$$\text{Current (it)} = I_m \sin (\omega t - \Phi)$$

Where Φ is leading phase angle.

Lagging Phase

When two waveforms of the same frequency are travelling along the same axis and one waveform is behind of another, then it is called 'Lagging phase wave'.

The voltage and current equations for leading phased waveforms are

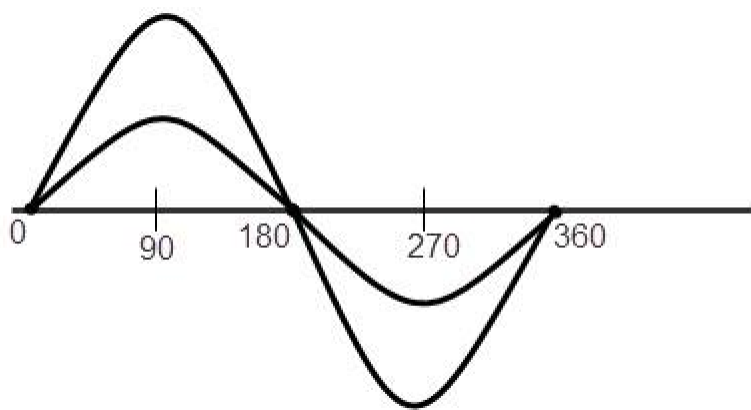
$$\text{Voltage (Vt)} = V_m \sin \omega t$$

$$\text{Current (it)} = I_m \sin (\omega t + \Phi)$$

Where Φ is lagging phase angle.

IN Phase Sine Waveforms

When the difference between phase of two alternating waves is zero, the waves are said to be -In-phase. This can be occurred when the two waveforms have the same frequency and the same phase. For in phase waveforms, the retardation is a whole number of wavelengths like 0, 1, 2, 3... The in-phase waveforms are shown in below figure.



The waveforms in the above picture have different amplitude (maximum voltage) but they have the same frequency.

Ex: If two sine waves A & B are out of phase and the phase difference is 25° then we can explain the relation between the waves as

Wave 'A' leads by wave 'B' by 25° or wave 'B' lags by wave 'A' by 25° . So the current and voltages of these waveforms also vary with the phase shift of the out of phased waveforms.

Voltage and Current Phase Relationships to R,L,C

The R L C circuit is also called as -Resonance circuit. The voltage and current behavior of the resistor, capacitor and inductors with respect to phase is explained below.

- Resistor: In resistor, the current and the voltage are in the same phase. So the phase difference between them is measured as 0.
- Capacitor: In a capacitor, the current and voltages are not in the same phase and the

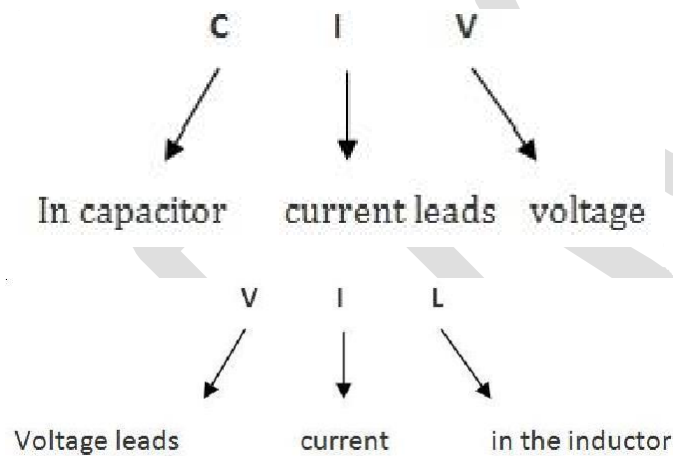
current leads voltage by 90° . So the phase difference between current and voltage in a capacitor is measured as 90° .

- Inductor: In inductor also, the current and voltages are not in the same phase. The voltage leads current by 90° . So the phase difference between voltage and current in capacitor is measured as 90° . This is exactly opposite to nature of the capacitor.

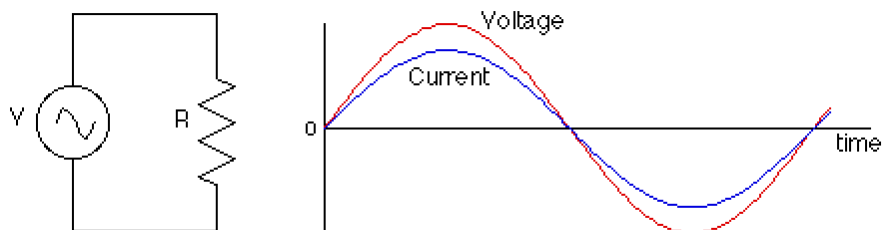
NOTE:

There is a simple technique to remember this voltage, current relationship, without any confusion. That technique is C I V I L

The first 3 letters C I V represents that, in a capacitor, I (current) leads V (voltage).



RESISTANCE IN AN AC CIRCUIT



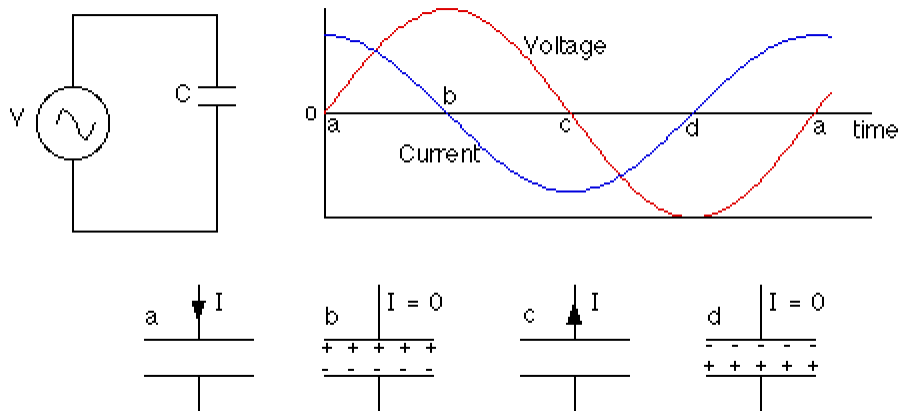
The relationship $V = IR$ applies for resistors in an AC circuit, so

$$I = V / R = (V_0 / R) \sin(\omega t) = I_0 \sin(\omega t)$$

In AC circuits we'll talk a lot about the phase of the current relative to the voltage. In a circuit

which only involves resistors, the current and voltage are in phase with each other, which means that the peak voltage is reached at the same instant as peak current. In circuits which have capacitors and inductors (coils) the phase relationships will be quite different.

CAPACITANCE IN AN AC CIRCUIT



Consider now a circuit which has only a capacitor and an AC power source (such as a wall outlet). A capacitor is a device for storing charging. It turns out that there is a 90° phase difference between the current and voltage, with the current reaching its peak 90° ($1/4$ cycle) before the voltage reaches its peak. Put another way, the current leads the voltage by 90° in a purely capacitive circuit.

To understand why this is, we should review some of the relevant equations, including:

relationship between voltage and charge for a capacitor: $CV = Q$

relationship between current and the flow of charge : $I = \Delta Q / \Delta t$

The AC power supply produces an oscillating voltage. We should follow the circuit through one cycle of the voltage to figure out what happens to the current.

Step 1 - At point a (see diagram) the voltage is zero and the capacitor is uncharged. Initially, the voltage increases quickly. The voltage across the capacitor matches the power supply voltage, so the current is large to build up charge on the capacitor plates. The closer the voltage gets to its peak, the slower it changes, meaning less current has to flow. When the voltage reaches a peak at point b, the capacitor is fully charged and the current is momentarily zero.

Step 2 - After reaching a peak, the voltage starts dropping. The capacitor must discharge now, so the current reverses direction. When the voltage passes through zero at point c, it's changing quite rapidly; to match this voltage the current must be large and negative.

Step 3 - Between points c and d, the voltage is negative. Charge builds up again on the capacitor

plates, but the polarity is opposite to what it was in step one. Again the current is negative, and as the voltage reaches its negative peak at point d the current drops to zero.

Step 4 - After point d, the voltage heads toward zero and the capacitor must discharge. When the voltage reaches zero it's gone through a full cycle so it's back to point a again to repeat the cycle.

The larger the capacitance of the capacitor, the more charge has to flow to build up a particular voltage on the plates, and the higher the current will be. The higher the frequency of the voltage, the shorter the time available to change the voltage, so the larger the current has to be. The current, then, increases as the capacitance increases and as the frequency increases.

Usually this is thought of in terms of the effective resistance of the capacitor, which is known as the capacitive reactance, measured in ohms. There is an inverse relationship between current and resistance, so the capacitive reactance is inversely proportional to the capacitance and the frequency:

A capacitor in an AC circuit exhibits a kind of resistance called capacitive reactance, measured in ohms. This depends on the frequency of the AC voltage, and is given by:

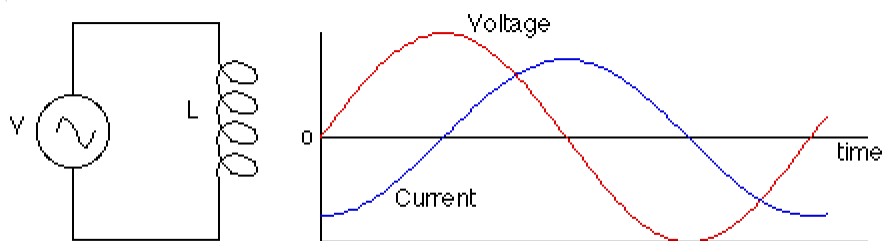
$$\text{capacitive reactance: } X_c = 1 / \omega C = 1 / 2\pi f C$$

We can use this like a resistance (because, really, it is a resistance) in an equation of the form $V = IR$ to get the voltage across the capacitor:

$$V = I X_c$$

Note that V and I are generally the rms values of the voltage and current.

INDUCTANCE IN AN AC CIRCUIT



An inductor is simply a coil of wire (often wrapped around a piece of ferromagnet). If we now look at a circuit composed only of an inductor and an AC power source, we will again find that there is a 90° phase difference between the voltage and the current in the inductor. This time, however, the current lags the voltage by 90° , so it reaches its peak $1/4$ cycle after the voltage

peaks.

The reason for this has to do with the law of induction:

$$\epsilon = -N \Delta\Phi / \Delta t \quad \text{or} \quad \epsilon = -L \Delta I / \Delta t$$

Applying Kirchoff's loop rule to the circuit above gives:

$$V - L \Delta I / \Delta t = 0 \quad \text{so} \quad V = L \Delta I / \Delta t$$

As the voltage from the power source increases from zero, the voltage on the inductor matches it. With the capacitor, the voltage came from the charge stored on the capacitor plates (or, equivalently, from the electric field between the plates). With the inductor, the voltage comes from changing the flux through the coil, or, equivalently, changing the current through the coil, which changes the magnetic field in the coil.

To produce a large positive voltage, a large increase in current is required. When the voltage passes through zero, the current should stop changing just for an instant. When the voltage is large and negative, the current should be decreasing quickly. These conditions can all be satisfied by having the current vary like a negative cosine wave, when the voltage follows a sine wave.

How does the current through the inductor depend on the frequency and the inductance? If the frequency is raised, there is less time to change the voltage. If the time interval is reduced, the change in current is also reduced, so the current is lower. The current is also reduced if the inductance is increased.

As with the capacitor, this is usually put in terms of the effective resistance of the inductor. This effective resistance is known as the inductive reactance. This is given by:

$$X_L = \omega L = 2\pi f L$$

where L is the inductance of the coil (this depends on the geometry of the coil and whether it got a ferromagnetic core). The unit of inductance is the henry.

As with capacitive reactance, the voltage across the inductor is given by:

$$V = IX_L$$

Where does the energy go?

One of the main differences between resistors, capacitors, and inductors in AC circuits is in what happens with the electrical energy. With resistors, power is simply dissipated as heat. In a capacitor, no energy is lost because the capacitor alternately stores charge and then gives it back again. In this case, energy is stored in the electric field between the capacitor plates. The amount

of energy stored in a capacitor is given by:

energy in a capacitor : Energy = $1/2 CV^2$

In other words, there is energy associated with an electric field. In general, the energy density (energy per unit volume) in an electric field with no dielectric is:

Energy density in an electric field = $1/2 \epsilon_0 E^2$

With a dielectric, the energy density is multiplied by the dielectric constant.

There is also no energy lost in an inductor, because energy is alternately stored in the magnetic field and then given back to the circuit. The energy stored in an inductor is:

energy in an inductor: Energy = $1/2 LI^2$

Again, there is energy associated with the magnetic field. The energy density in a magnetic field is:

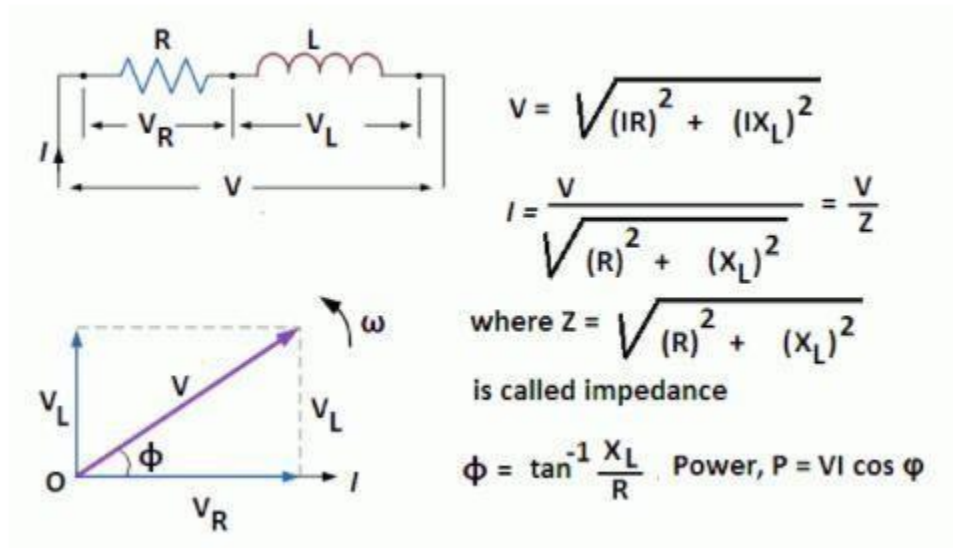
Energy density in a magnetic field = $B^2 / (2 \mu_0)$

RC | RLC | RL Series Circuits

In actual practice, AC circuits contain two or more than two components connected in series. In a series circuit, each component carries the same current. An AC series circuit may be classified as under:

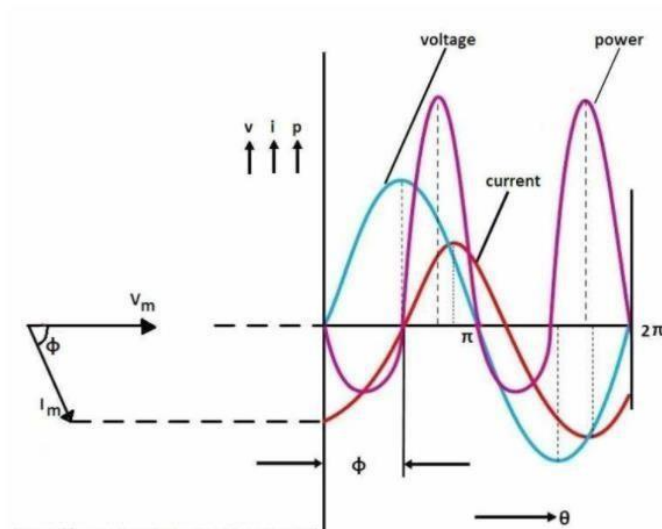
- RL series circuit
- RC series circuit
- RLC series circuit

RL SERIES CIRCUIT



In an RL series circuit, a pure resistance (R) is connected in series with a coil having the pure inductance (L). To draw the phasor diagram of RL series circuit, the current I (RMS value) is taken as reference vector because it is common to both elements.

Voltage drop V_R is in phase with current vector, whereas, the voltage drop in inductive reactance V_L leads the current vector by 90° since current lags behind the voltage by 90° in the purely inductive circuit. The vector sum of these two voltage drops is equal to the applied voltage V (RMS value).



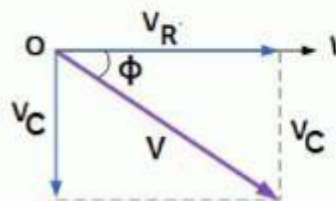
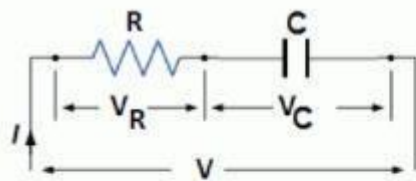
The power waveform for RL series circuit is shown in the figure. In this figure, voltage wave is considered as a reference. The points for the power waveform are obtained from the product of

the corresponding instantaneous values of voltage and current.

It is clear from the power waveform that power is negative between 0 and ϕ and between 180° and $(180^\circ + \phi)$. The power is positive during rest of the cycle.

Since the area under the positive loops is greater than that under the negative loops, the net power over a complete cycle is positive. Hence a definite quantity of power is consumed by the RL series circuit. But power is consumed in resistance only; inductance does not consume any power.

RC SERIES CIRCUIT



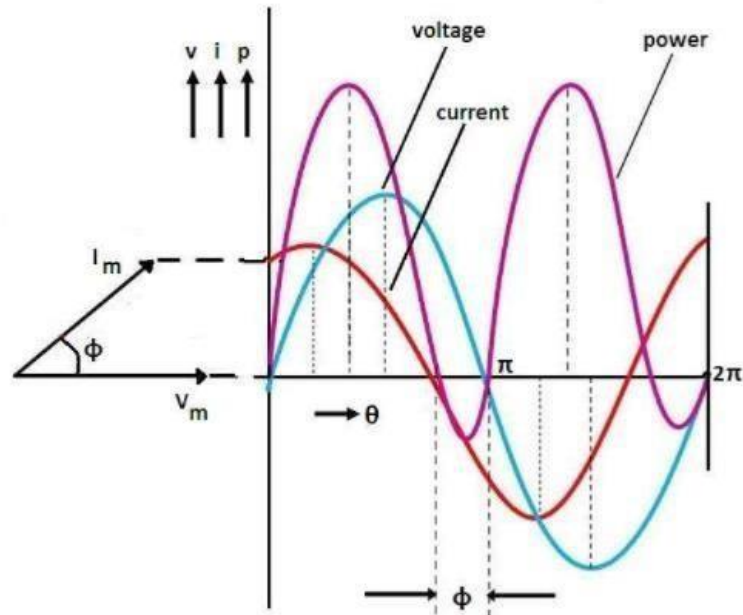
$$V = \sqrt{(IR)^2 + (IX_C)^2}$$

$$I = \frac{V}{\sqrt{(R)^2 + (X_C)^2}} = \frac{V}{Z}$$

where $Z = \sqrt{(R)^2 + (X_C)^2}$
is called impedance

$$\phi = \tan^{-1} \frac{X_C}{R} \quad \text{Power, } P = VI \cos \phi$$

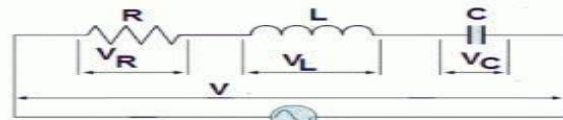
In an RC series circuit, a pure resistance (R) is connected in series with a pure capacitor (C). To draw the phasor diagram of RC series circuit, the current I (RMS value) is taken as reference vector. Voltage drop V_R is in phase with current vector, whereas, the voltage drop in capacitive reactance V_C lags behind the current vector by 90° , since current leads the voltage by 90° in the pure capacitive circuit. The vector sum of these two voltage drops is equal to the applied voltage V (RMS value).



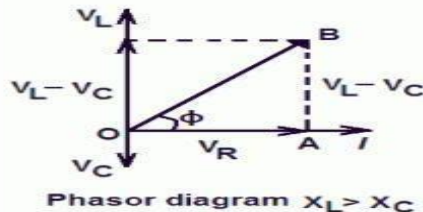
The power waveform for RC series circuit is shown in the figure. In this figure, voltage wave is considered as a reference. The points for the power waveform are obtained from the product of the corresponding instantaneous values of voltage and current. It is clear from the power waveform that power is negative between $(180^\circ - \phi)$ and 180° and between $(360^\circ - \phi)$ and 360° . The power is positive during rest of the cycle.

Since the area under the positive loops is greater than that under the negative loops, the net power over a complete cycle is positive. Hence a definite quantity of power is consumed by the RC series circuit. But power is consumed in resistance only; capacitor does not consume any power.

RLC SERIES CIRCUIT



RLC Series Circuit



Phasor diagram $X_L > X_C$

$$V = \sqrt{(IR)^2 + I^2(X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{(R)^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

$$\text{where } Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

is called impedance

$$\phi = \tan^{-1} \frac{X}{R} \quad \text{Power, } P = VI \cos \phi$$

$$\text{where } X = X_L - X_C$$

In an RLC series circuit a pure resistance (R), pure inductance (L) and a pure capacitor (C) are connected in series. To draw the phasor diagram of RLC series circuit, the current I (RMS value) is taken as the reference vector. The voltages across three components are represented in the phasor diagram by three phasors V_R , V_L and V_C respectively.

The voltage drop V_L is in phase opposition to V_C . It shows that the circuit can either be effectively inductive or capacitive. In the figure, phasor diagram is drawn for the inductive circuit. There can be three cases of RLC series circuit.

- When $X_L > X_C$, the phase angle ϕ is positive. In this case, RLC series circuit behaves as an RL series circuit. The circuit current lags behind the applied voltage and power factor is lagging. In this case,

if the applied voltage is represented by the equation;

$v = V_m \sin \omega t$ then, the circuit current will be represented by the equation;

$$i = I_m \sin (\omega t - \phi).$$

- When $X_L < X_C$, the phase angle ϕ is negative. In this case, the RLC series circuit behaves as an RC series circuit. The circuit current leads the applied voltage and power factor is leading. In this case, the circuit current will be represented by the equation:

$$i = I_m \sin (\omega t + \phi).$$

- When $X_L = X_C$, the phase angle ϕ is zero. In this case, the RLC series circuit behaves like a purely resistive circuit. The circuit current is in phase with the applied voltage and

power factor is unity. In this case, the circuit current will be represented by the equation:

$$i = I_m \sin (\omega t).$$

Example

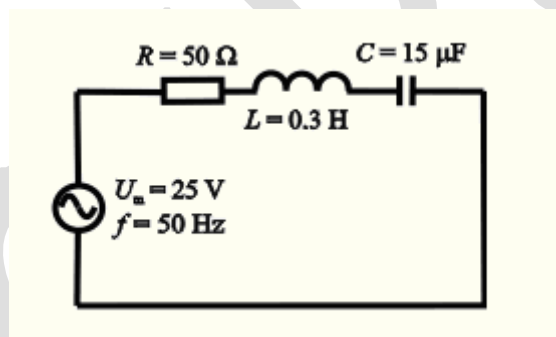
In AC circuit is composed of a serial connection of:

a resistor with resistance 50Ω ,

a coil with inductance 0.3 H

and a capacitor with capacitance $15 \mu\text{F}$.

The circuit is connected to an AC voltage source with amplitude 25 V and frequency 50 Hz . Determine the amplitude of electric current in the circuit and a phase difference between the voltage and the current.



We write down the quantities given in the assignment:

Resistance of resistor $R = 50 \Omega$

Inductance of the coil $L = 0.3 \text{ H}$

Capacitance $C = 15 \mu\text{F} = 15 \cdot 10^{-6} \text{ F}$

Amplitude of AC voltage source $U_m = 25 \text{ V}$

Frequency of source $f = 50 \text{ Hz}$

Resistor, coil and capacitor are connected in series.

Quantities that we want to determine:

Amplitude of the current in the circuit $I_m = ? (\text{A})$

Phase difference between the voltage and the current in the circuit $\varphi = ? (^\circ)$

How to solve this task:

1. We evaluate the current amplitude. We apply Ohm's law for the alternating current, which expresses the relationship between an overall impedance Z , a voltage amplitude of the source U_m and a current amplitude I_m . For this calculation, we know all the quantities from assignment.
2. Electric current flowing through all the components connected in series is of the same size; however the voltage on the components is out of phase with the current. To obtain the phase difference (shift) between voltage and current we use a phasor diagram.

A phasor is an "arrow" that we use to plot the current and voltage values on individual components of the circuit into a phasor diagram. Its magnitude reflects the amplitude of the voltage or current, and its direction indicates the phase angle.

Drawing a phasor diagram for a series circuit:

We plot the values of voltage and current on individual components in the AC circuit into the phasor diagram.

The current is of the same size on all the components, the phasor of current I_m is therefore the same for all the components and is usually drawn in the positive direction of the x-axis.

The phasor of voltage is on the resistor U_R parallel to the current phasor, because the phase difference between the voltage and current is zero – in this case voltage and current are in phase. In the figure the phasor is illustrated by green.

The voltage on the coil U_L leads the current by $\pi/2$ (quarter of a period), therefore we draw its phasor pointing upwards – in the positive direction of the imaginary y-axis. We consider the fact that the phasors rotate in a counter clockwise direction. In the figure, this phasor is represented by yellow.

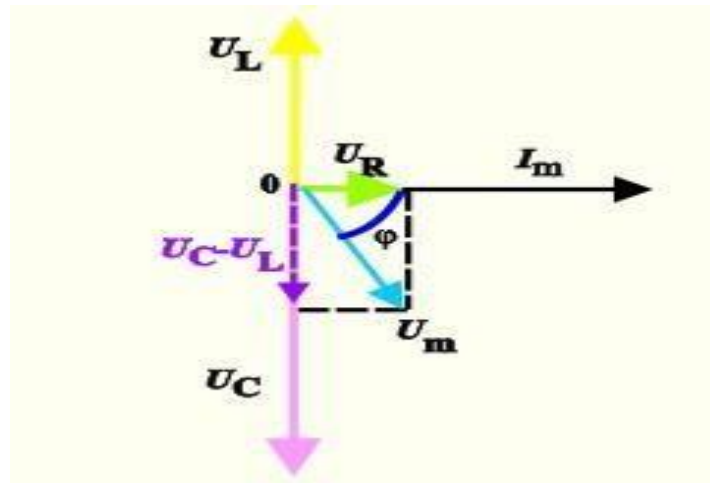
The current on the capacitor leads the voltage U_C by $\pi/2$. Therefore we draw the phasor pointing downward – that is in negative direction of the y-axis. This phasor is represented by pink.

The amplitude of the overall voltage is obtained by a "vector sum" of phasors of the voltage on individual components. First, we subtract the voltage on the capacitor U_C from the voltage on the coil U_L (in the picture drawn in purple). Then we add this vector and the vector of the voltage on the resistor U_R . The phasor of the voltage amplitude of the entire circuit is represented by light blue.

A phase difference between the voltage and the current is said to be the angle φ between the

current phasor and the overall voltage phasor. The angle ϕ is drawn by navy blue

For an RLC circuit and the given quantities the phasor diagram looks like this:



Phasor diagram

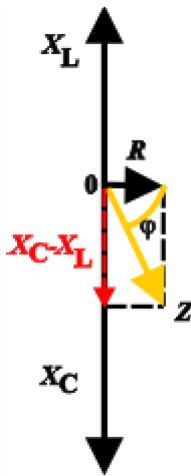
In the following figures the phasor diagrams are not illustrated by the same colour

To get the total impedance Z from the phasor diagram, instead of the voltage on individual components in the phasor diagram we plot the inductance X_L , the capacitance X_C and the resistance R .

From Ohm's law we know that:

$$U_C = I_m X_C, \quad U_L = I_m X_L, \quad U_R = I_m R.$$

Since the current through all the components is the same, the impedances of individual elements are proportional to the voltage, so we can draw a diagram similar to the voltage phasors.



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To calculate the impedance Z we use the rectangular triangle we can see in the phasor diagram. The impedance Z is evaluated by using Pythagorean theorem.

$$Z^2 = R^2 + (X_C - X_L)^2$$

or:

$$Z^2 = R^2 + (X_L - X_C)^2.$$

The difference between the relationships is whether the current leads voltage or the voltage leads the current. The size of the impedance Z is however not affected

By substituting the relations of inductance and capacitance we obtain:

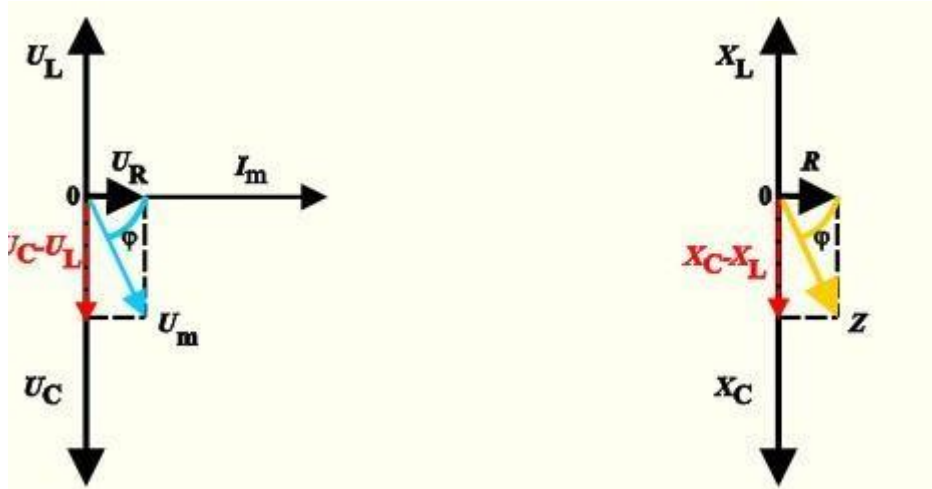
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The formula for expressing the impedances Z from Ohm's law is:

$$Z = \frac{U_m}{I_m} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

We determine the amplitude of the current I_m . We can easily derive

$$I_m = \frac{U_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



The phase shift is expressed from the phasor diagram usually in the form:

$$\operatorname{tg} \varphi = \frac{U_L - U_C}{U_R} = \frac{I_m \omega L - \frac{I_m}{\omega C}}{I_m R} = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{X_L - X_C}{R}.$$

When we draw a phasor diagram and a phase difference, the formula

$$\operatorname{tg} \varphi = \frac{U_L - U_C}{U_R} = \frac{X_L - X_C}{R}$$

can be replaced by the formula

$$\operatorname{tg} \varphi = \frac{U_C - U_L}{U_R} = \frac{X_C - X_L}{R}.$$

We must be careful in interpreting the results. In the first case the numerator says that we consider the case when the voltage leads the current (similarly as on the coil). In the second case, on the other hand, the current leads the voltage. We choose a suitable relationship either from the phasor diagram, where we can see the phase difference between the voltage and current, or we choose one of the relations and interpret the result through the sign of the resulting value. If you choose, for example, the second formula for expressing the phase difference and the resulting value has a plus sign, then the current leads the voltage. However, if the resulting value of the

phase difference is negative, then the voltage leads the current.

The current amplitude:

$$I_m = \frac{U_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} =$$
$$= \frac{25}{\sqrt{50^2 + \left(2\pi \cdot 50 \cdot 0.3 - \frac{1}{2\pi \cdot 50 \cdot 15 \cdot 10^{-6}}\right)^2}} \text{ A} \doteq 0.2 \text{ A}$$

We can evaluate the **phase difference** by the **impedance**:

$$\operatorname{tg} \varphi = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{2\pi \cdot 50 \cdot 0.3 - \frac{1}{2\pi \cdot 50 \cdot 15 \cdot 10^{-6}}}{50} \doteq -2.4$$

Or it can be evaluated by using the **voltage** on individual components in the circuit:

The voltage on individual components of the circuit is:

$$U_R = I_m R \doteq 0.2 \cdot 50 \text{ V} = 10 \text{ V}$$

$$U_L = I_m \omega L \doteq 0.2 \cdot 2\pi \cdot 50 \cdot 0.3 \text{ V} \doteq 18.85 \text{ V}$$

$$U_C = \frac{I_m}{\omega C} \doteq \frac{0.2}{2\pi \cdot 50 \cdot 15 \cdot 10^{-6}} \text{ V} \doteq 42.44 \text{ V}$$

We determine the size of the phase difference between the voltage and the current in the circuit from the phase diagram:

$$\operatorname{tg} \varphi = \frac{U_L - U_C}{U_R} \doteq \frac{18.85 - 42.44}{10} \doteq -2.4$$

Activ
Go to

Both methods gave us the same result:

In the series RLC circuit the amplitude of the current is approximately:

$$I_m = 0.2 \text{ A.}$$

The phase difference between the voltage and the current is about:

$$\varphi = -67^\circ.$$

The sign of the phase difference means that the current leads the voltage by about 67° (since the current leads the voltage, this circuit acts as a capacitor).

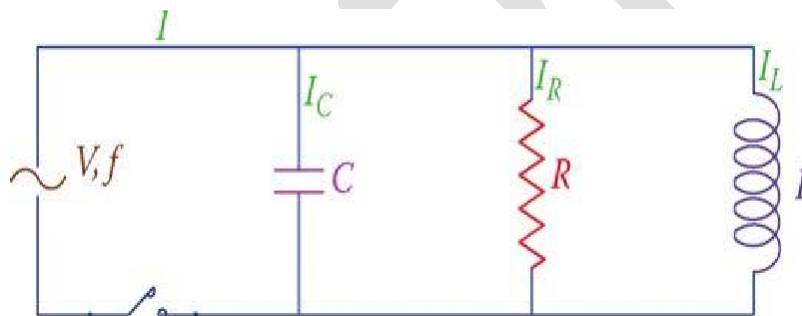
Parallel RLC Circuit

In parallel RLC circuits the three basic components are in parallel with each other, and, therefore, all are subject to the same voltage. The current for each branch, however, depends on the impedance of the branch and can be individually determined by employing **Ohm's law**.

For a parallel RLC circuit, the voltage is common for all the three types of components because it is the **same voltage** that is applied to each component. Nevertheless, the **currents in the three branches are not in phase with each other**. This means that the currents in the three branches do not simultaneously reach their peak values or zero values.

Hence, the total current cannot be determined by algebraically adding the individual values of the currents in the resistor, inductor, and capacitor.

A parallel RLC circuit is shown in **Figure 1**. As in the case of series RLC circuits, we need to find the total current and the power consumption for the whole circuit or for each individual branch.



For this circuit the voltage applied to each component in each branch is the same. Therefore, the current in each component can be found from dividing the voltage by the branch impedance. Then the currents can be added together.

However, because the currents in the three components are not in phase with each other (they do not reach their maximum and minimum values at the same time), they cannot be algebraically added together and must be added in vector form.

Figure 1 illustrates the vector representation of the three currents in a typical parallel RLC circuit. It shows that the current in the resistor is in phase with the applied voltage, the current in the capacitor leads the applied voltage (remember ICE) and the current in the inductor lags the voltage (remember ELI).

Furthermore, note that for this vector representation of the currents and voltage in a parallel RLC circuit, because the voltage is the common variable for all branches, you start by drawing the vector for the voltage as the reference vector. (In series RLC circuit you started this process by drawing the vector for the current.)

To find the total current in a parallel RLC circuit, one needs to find the vector sum of the currents

in R , L , and C .

Because the current in the inductor and the current in the capacitor are 180° out of phase, in adding them together their values are subtracted from each other. Thus, the relationship for the total current of the circuit, I , and the individual component currents I_R , I_L , and I_C is

$$I = \sqrt{I_R^2 + (I_L - I_C)^2} \quad (1)$$

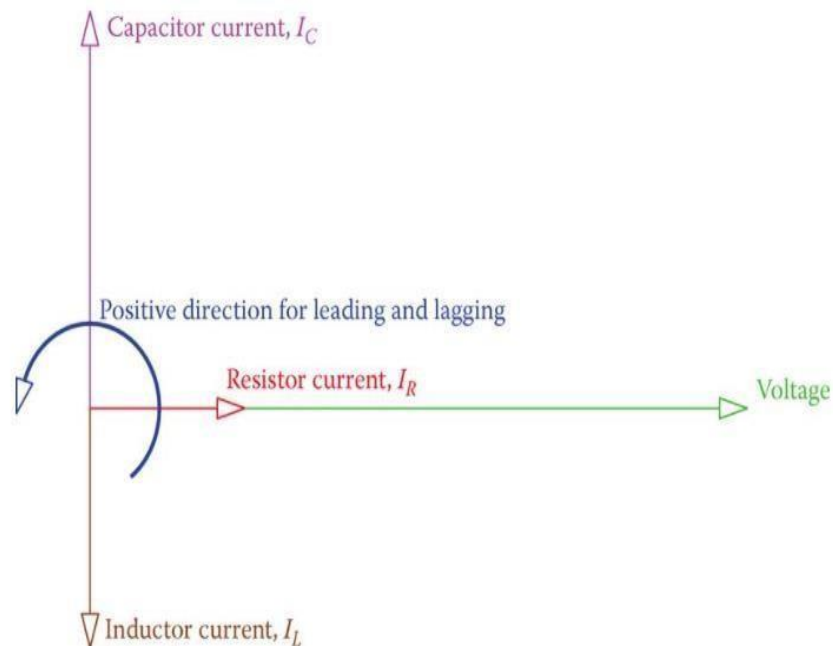


Figure 2 Vectors for the voltage and the three different currents in the RLC parallel circuit.

Parallel RLC Circuit Example 1

In the circuit shown in **Figure 3** the current is 1.8 A. If the current through the capacitor is 1.5 A, find the applied voltage and the resistance of the resistor.

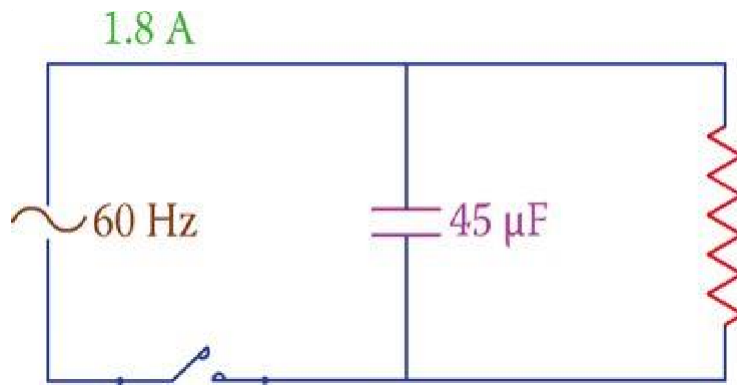


Figure 3 Circuit corresponding to **Example 1**.

Solution

For 60 Hz frequency, the reactance of the capacitor is

$$X_C = \frac{1}{2 * 3.14 * 60 * 0.000045} = 59\Omega$$

Thus, the applied voltage is

$$59 * 1.5 = 88.5V$$

Because this circuit has no inductor, the value of L in **Equation 1** is set to zero and the result is

$$I = \sqrt{I_R^2 + I_C^2}$$

Which leads to

$$I_R = \sqrt{1.8^2 - 1.5^2} = 0.995 = 1A$$

And the resistance of the resistor is

$$88.5 \div 1 = 88.5\Omega$$

If in **Equation 1**, the values for I_R , I_L , and I_C are replaced by $\frac{V}{R}$, $\frac{V}{X_L}$ and I is written as the ratio of the applied voltage to the circuit impedance Z , we have

$$\frac{V}{Z} = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L}\right)^2}$$

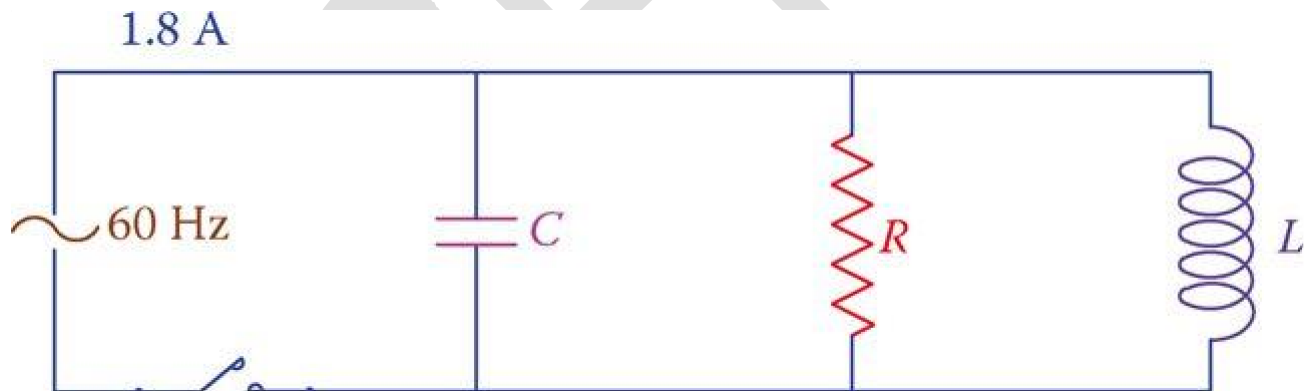
By omitting V from both sides the relationship between Z and R , L , and C can be found then as

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2} \quad (2)$$

Equation 2 can be used to find the equivalent impedance of the three components in parallel. The circuit current can also be found this way by dividing the applied voltage by Z or by directly multiplying $1/Z$ by the applied voltage.

Parallel RLC Circuit Example 2

In the circuit shown in **Figure 4**, $R = 55 \Omega$, $L = 0.08 \text{ H}$, and $C = 1 \mu\text{F}$, find the impedance of the circuit and the applied voltage.



$$X_L = 2 * 3.14 * 60 * 0.08 = 30.16\Omega$$

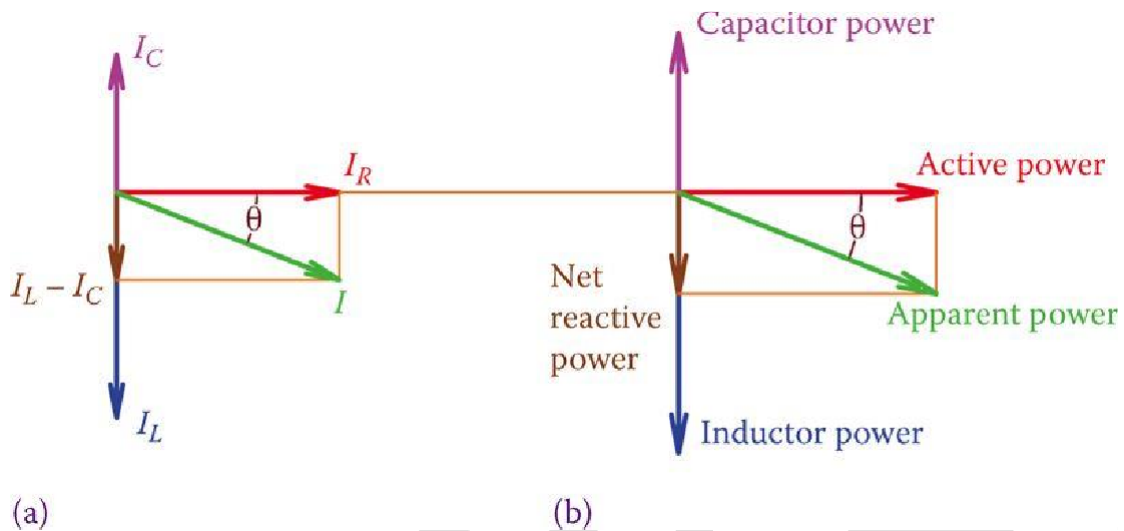
$$X_C = \frac{1}{2 * 3.14 * 60 * 0.000001} = 26.5\Omega$$

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{55}\right)^2 + \left(\frac{1}{55} - \frac{1}{26.5}\right)^2} = \frac{1}{53.33}$$

$$Z = 53.33\Omega$$

$$\text{Applied voltage} = V = ZI = (53.33) (1.8) = \mathbf{96 \text{ V.}}$$

Equation 2 also implies that the value for Z is smaller than R for parallel RLC circuits. A vector representation of I_R , I_L , I_C , and I is shown in **Figure 5**, which also shows the powers in the three components and the apparent power.



Reactive power is the vector sum of the inductive and capacitive powers. Depending on if inductive power (Q_L) or the capacitive power (Q_C) is larger the vectors for I and the apparent power S fall below or above the horizontal reference. The former implies that the current leads the voltage and the latter denotes that the current lags the voltage.

Because in practice the majority of applications (including home and industrial circuits) are parallel circuits, any circuit is categorized to be leading or lagging. If in a circuit the current leads the voltage, the circuit is said to be leading; if the current lags the voltage, the circuit is said to be lagging.

Power Factor in Parallel RLC Circuits

Figure 5 shows a lagging circuit. In practice, most of the circuits are lagging because of the presence of electric motors, unless the effects of electric motors are compensated by inserting capacitors that introduce capacitive power to a circuit (see power factor correction). The power factor in a parallel RLC circuit is determined from

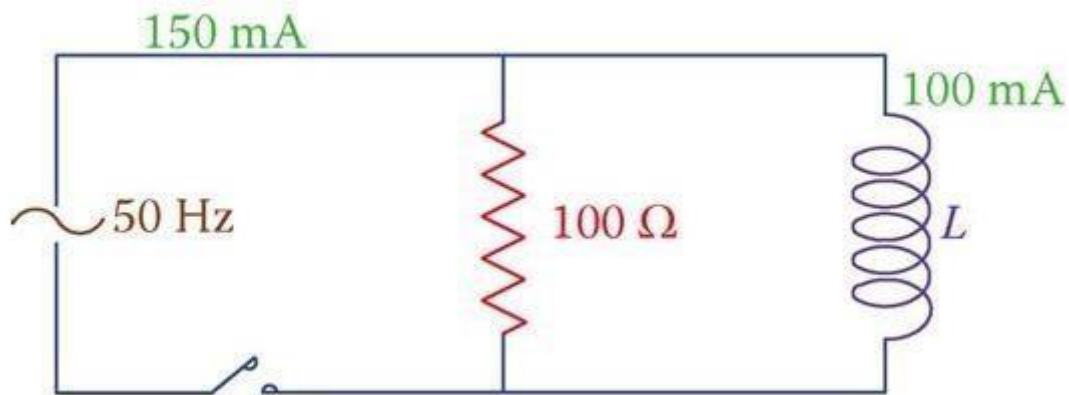
$$pf = \frac{Z}{R} = \frac{I_R}{I} = \frac{\text{Active Power}}{\text{Apparent Power}} \quad (3)$$

Note that the power factor by itself is not sufficient to describe a circuit. It has to be accompanied by the statement for leading or lagging. A circuit may have the same power factor in two cases, either leading or lagging. Sometimes the leading or lagging is attributed to the

power factor. For example, one may say a circuit has a leading power factor of 0.90.

Parallel RLC Circuit Example 3

In the circuit shown in Figure 6, the total current is 150 mA and the current through the inductor is 100 mA. Determine what the applied voltage is. Also, knowing that the frequency is 50 Hz, find the value of L .



Solution

The applied voltage can be found by multiplying the resistor current by 100 Ω. Having only a resistor and an inductor in this circuit **Equation 1** leads to

$$I_R = \sqrt{I^2 - I_L^2} = \sqrt{150^2 - 100^2} = 0.1118A$$

$$V = 100 * 0.0008 = 11.18V$$

$$X_L = 11.18 \div 0.100 = 111.8\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{111.8}{2\pi * 50} = 35.6mH$$

In a parallel AC circuit, if the current leads the voltage, the circuit is said to be leading; if the current lags, the voltage the circuit is said to be lagging.

TYPES OF AC POWER

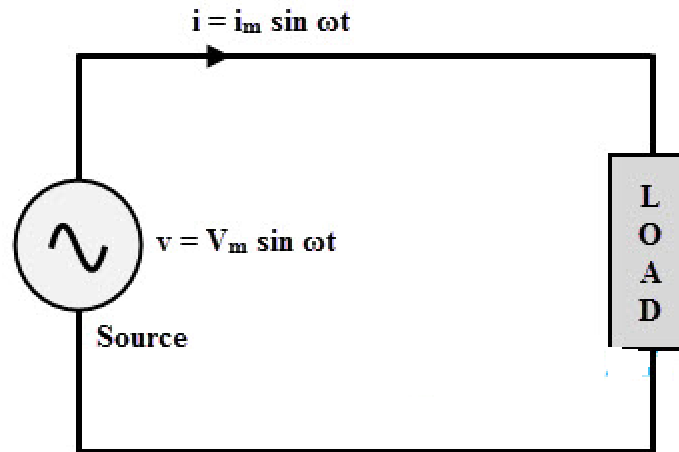
The power in any electrical circuit can be obtained by performing multiplication of voltage and current values in that circuit. This is applicable for both DC and AC circuits.

i.e., power = (Current value) x (Voltage value)

$$P = V \times I$$

Power is measured in watts. In DC circuits and pure AC circuits without any non linear components, the current and voltage waveforms are 'in phase'.

So the power at any instant of time in that circuit is obtained by multiplying the voltage and current. However, in case of AC circuits, this will not be so (as mentioned above the existence of phase shift).



Consider the above circuit in which AC supply is given to a load. Voltages and currents in the circuit are given as

$$v = V_m \sin \omega t \Rightarrow v = \sqrt{2} V \sin \omega t$$

$$i = I_m \sin \omega t \Rightarrow i = \sqrt{2} I \sin (\omega t \pm \phi)$$

Where $V (= V_m/\sqrt{2})$ and $I (= I_m/\sqrt{2})$ are the RMS values of applied voltage and current flowing through the circuit respectively. Φ is the phase difference between voltage and current, to which + sign indicates the leading phase angle while negative indicates the lagging phase angle.

Then the instantaneous power delivered to load by source is given by,

$$p = vi = 2 VI \sin \omega t \sin (\omega t \pm \phi)$$

$$= VI (\cos \phi - \cos (2\omega t \pm \phi))$$

$$p = VI \cos \phi (1 - \cos 2\omega t) \pm VI \sin \phi \sin 2\omega t$$

The above power equation consists of two terms, namely

1. A term proportional to $VI \cos \phi$ which is pulsating around the average value of $VI \cos \phi$
2. A term proportional to $VI \sin \phi$ pulsating at twice the supply frequency, producing an average of zero over a cycle.

So there are 3 forms of powers in AC circuits. They are

1. Active power or True power or Real power
2. Reactive power
3. Apparent power

Active Power

The actual amount of power being dissipated or performs the useful work in the circuit is called as active or true or real power. It is measured in watts, practically measured in KW (kilowatts) and MW (megawatts) in power systems.

It is denoted by the letter P (capital) and it is equal to the average value of $p = VI \cos \phi$. It is the desired outcome of an electrical system which drives the circuit or load.

$$P = VI \cos \phi$$

Reactive Power

The average value of the second term in the above derived expression is zero, so the power contributed by this term is zero. The component, which is proportional to $VI \sin \phi$ is called as reactive power, represented by the letter Q.

Even though it is a power, but not measured in watts as it is a non active power and hence, it is measured in Volt-Amperes- Reactive (VAR). The value of this reactive power can be negative or positive depends on the load power factor.

This is because inductive load consumes the reactive power while capacitive load generates the reactive power.

$$Q = VI \sin \phi$$

Significance of Reactive Power

The reactive power is one of the total power components that travel back and forth in the circuit or line. It can be termed as the rate of change of energy with respect to time that keeps on flowing from source to reactive components during positive half cycle and back to the components from source during negative cycle. Therefore, it never gets consumed by the load.

In the normal sense, this fictitious power is not at all a power but only a power-like measure of reactive component of the current. If there exist excess amount of reactive power, power factor is greatly reduced. This low power factor is undesirable in terms of operating efficiency and operational costs.

And also this power causes to draw additional current from the supply leads to additional losses and greater capacity of the equipments. That's why this power has been referred as the cholesterol of power lines in a joking manner.

In order to minimize losses and to increase the capacity of the available equipment, utility companies make use VAR compensation techniques or power factor correction equipments. Generally, these reactive compensation techniques are implemented at the load side.

However, this reactive power is useful for generating necessary magnetic fields for operation of inductive devices like transformers, AC motors, etc. It also helps to regulate the voltage in heavy power supply mechanisms.

Apparent Power

The complex combination of true or active power and reactive power is called apparent power. Without reference to any phase angle, the product of voltage and current gives the apparent power. The apparent power is useful for rating the power equipment.

It can also be expressed as the square of the current multiplied by the circuit's impedance. It is denoted by the letter S and measured in Volt-Amperes (VA), practical units include KVA (Kilo volt-amperes) and MVA (mega volt-amperes).

Apparent power = RMS voltage \times RMS current

Apparent power, $S = V \times I$

In complex form, $S = V I^*$

$S = V \angle 0^\circ I \angle \phi$ (for lagging load

current) $S = V I \angle \phi$

$S = V I \cos \phi + jV I \sin \phi$

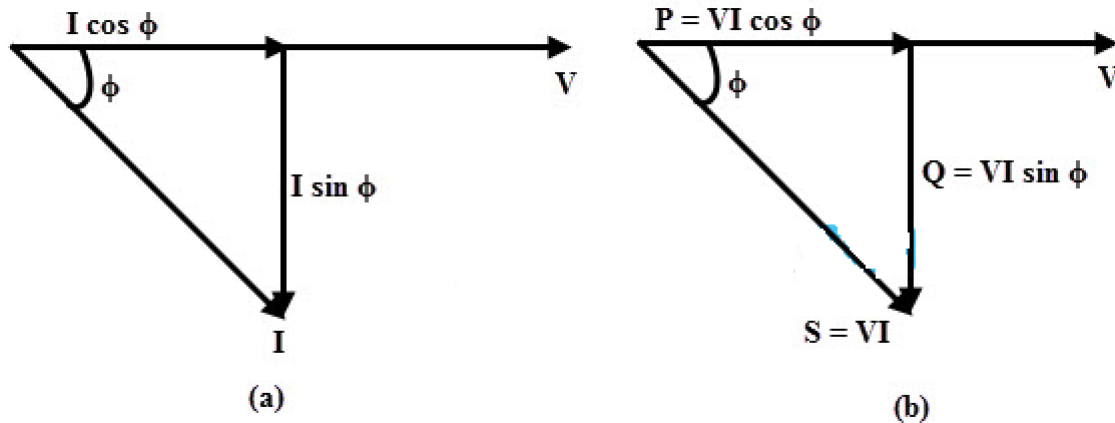
$= P + jQ$

Or $S = I^2 Z$

Power Triangle

The relation between active, reactive and apparent power can be expressed by representing

quantities as vectors, which is also called as power triangle method as shown below. In this phasor diagram voltage is considered as reference vector. The voltage & current phasor diagram is the basis for the formation of the power triangle.



In figure (a), current lags the applied voltage by angle ϕ . The horizontal component of the current is $I \cos \phi$ and the vertical component of the current is $I \sin \phi$. If each of the current phasor is multiplied by the voltage V , the power triangle is obtained as shown in the figure (b).

The active power is contributed by the component $I \cos \phi$ in phase with voltage while reactive power is produced by the quadrature component.

Therefore, the apparent power or the hypotenuse of the triangle is obtained by combining real and reactive power vectorially.

Using Pythagoras's theorem, the sum of squares of the two adjacent sides (active power and reactive power) is equal to the square of the diagonal (apparent power). i.e.,

$$(\text{Apparent power})^2 = (\text{Real Power})^2 + (\text{Reactive Power})^2$$

$$S^2 = P^2 + Q^2$$

$$S = \sqrt{(Q^2 + P^2)}$$

Where

S = apparent power measured in kilovolt amps, kVA

Q = reactive power measured in kilovolt amps reactive, kVAR

P = active power measured in kilowatts, kW

In terms of resistive, inductive and impedance elements, the power forms can be expressed as

$$\text{Active power} = P = I^2R$$

$$\text{Reactive power} = Q = I^2X$$

$$\text{Apparent power} = S = I^2Z$$

Where

X is inductance

Z is impedance.

Power Factor

The power factor is the cosine angle between the voltage and current. The power factor can be expressed in terms of the above discussed power forms. Consider the power triangle in above figure in which power factor is the ratio of active power to apparent power. Power factor defines the efficiency of the circuit.

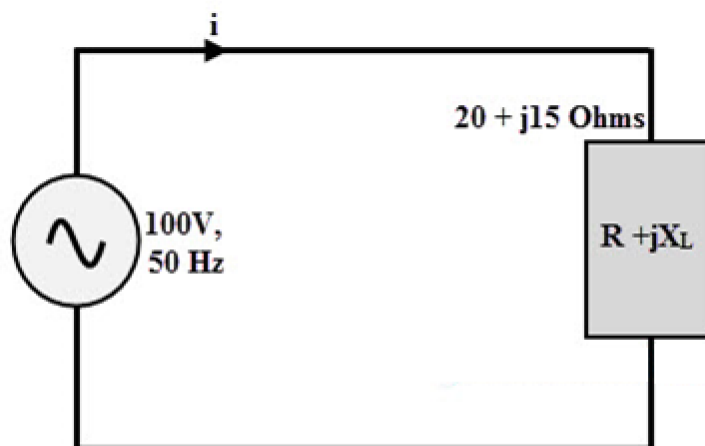
$$\text{Power factor (PF)} = (\text{Active power in watts})/(\text{Apparent power in volt amps})$$

$$\text{PF} = VI \cos \phi / VI$$

$$\text{PF} = \cos \phi$$

Example Problem

If an AC power supply of 100V, 50Hz is connected across a load of impedance, $20 + j15$ Ohms. Then calculate the current flowing through the circuit, active power, apparent power, reactive power and power factor.



Given that, $Z = R + jXL = 20 + j 15 \Omega$

Converting the impedance to polar form, we get

$$Z = 25 \angle 36.87^\circ \Omega$$

Current flowing through the circuit,

$$I = V/Z = 100 \angle 0^\circ / 25 \angle 36.87^\circ$$

$$I = 4 \angle -36.87^\circ$$

Active power, $P = I^2 R = 4^2 \times 20 = 320$ watts

$$\text{Or } P = VI \cos \phi = 100 \times 4 \times \cos (36.87^\circ) = 320.04 \approx 320 \text{ W}$$

Apparent power, $S = VI = 100 \times 4 = 400$ VA

$$\text{Reactive power, } Q = \sqrt{(S^2 - P^2)}$$

$$= \sqrt{(400^2 - 320^2)} = 240 \text{ VAR}$$

Power factor, $\text{PF} = \cos \phi = \cos 36.87^\circ = 0.80$ lagging.

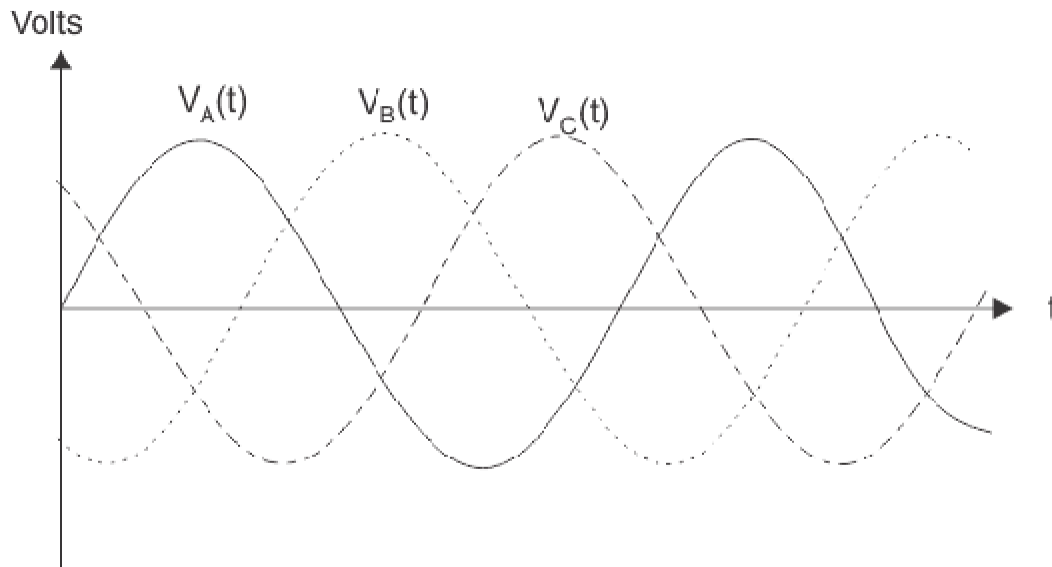
THREE PHASE CIRCUITS

There are two types of system available in electric circuit, single phase and **three phase system**. In single phase circuit, there will be only one phase, i.e the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase. This is an old system using from previous time.

In 1882, new invention has been done on polyphase system, that more than one phase can be used for generating, transmitting and for load system. **Three phase circuit** is the polyphase system where three phases are send together from the generator to the load.

Each phase are having a phase difference of 120° , i.e 120° angle electrically. So from the total of 360° , three phases are equally divided into 120° each. The power in **three phase system** is continuous as all the three phases are involved in generating the total power. The sinusoidal waves for 3 phase system is shown below-

The three phases can be used as single phase each. So if the load is single phase, then one phase can be taken from the **three phase circuit** and the neutral can be used as ground to complete the circuit.



Why Three Phase is Preferred Over Single Phase?

There are various reasons for this question because there are numbers of advantages over single phase circuit. The three phase system can be used as three single phase line so it can act as three single phase system. The three phase generation and single phase generation is same in the generator except the arrangement of coil in the generator to get 120° phase difference. The conductor needed in three phase circuit is 75% that of conductor needed in single phase circuit. And also the instantaneous power in single phase system falls down to zero as in single phase we can see from the sinusoidal curve but in three phase system the net power from all the phases gives a continuous power to the load.

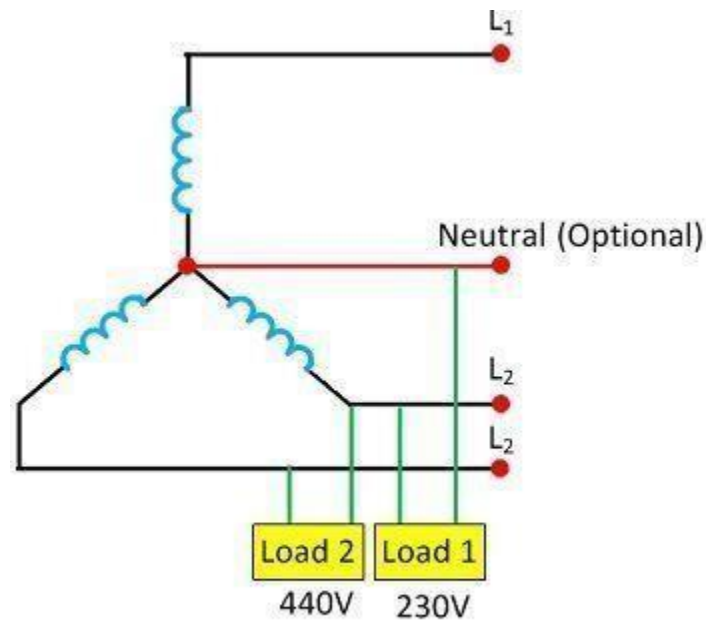
Till now we can say that there are three voltage source connected together to form a three phase circuit and actually it is inside the generator. The generator is having three voltage sources which are acting together in 120° phase difference. If we can arrange three single phase circuit with 120° phase difference, then it will become a three phase circuit. So 120° phase difference is must otherwise the circuit will not work, the three phase load will not be able to get active and it may also cause damage to the system.

The size or metal quantity of three phase devices is not having much difference. Now if we consider the transformer, it will be almost same size for both single phase and three phase because transformer will make only the linkage of flux. So the three phase system will have higher efficiency compared to single phase because for the same or little difference in mass of transformer, three phase line will be out whereas in single phase it will be only one. And losses will be minimum in three phase circuit. So overall in conclusion the three phase system will have better and higher efficiency compared to the single phase system. In three phase circuit, connections can be given in two types:

1. Star connection
2. Delta connection

Star Connection

The star connection requires four wires in which there are three phase conductors and one neutral conductor. Such type of connection is mainly used for long distance transmission because it has a neutral point. The neutral point passes the unbalanced current to the earth and hence make the system balance.



3 - phase Star Connected System Circuit Globe

The star connected three phase systems gives two different voltages, i.e., the 230 V and 440V. The voltage between the single phase and the neutral is 230V, and the voltage between the two phases is equal to the 440V.

Voltage and Current Values in Three-Phase star Systems

When we measure voltage and current in three-phase systems, we need to be specific as to *where* we're measuring.

Line voltage refers to the amount of voltage measured between any two line conductors in a balanced three-phase system. With the above circuit, the line voltage is roughly 208 volts.

Phase voltage refers to the voltage measured across any one component (source winding or

load impedance) in a balanced three-phase source or load.

For the circuit shown above, the phase voltage is 120 volts. The terms *line current* and *phase current* follows the same logic: the former referring to the current through any one line conductor, and the latter to the current through any one component.

Y-connected sources and loads always have line voltages greater than phase voltages, and line currents equal to phase currents. If the Y-connected source or load is balanced, the line voltage will be equal to the phase voltage times the square root of 3:

For "Y" circuits:

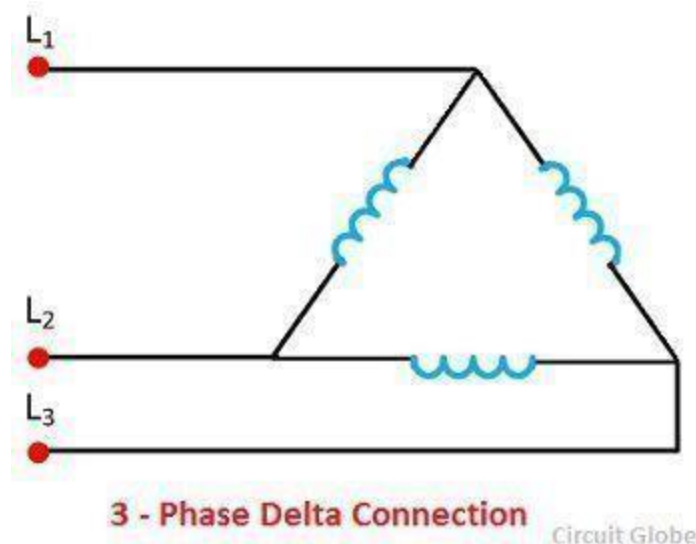
$$E_{\text{line}} = \sqrt{3} E_{\text{phase}}$$

$$I_{\text{line}} = I_{\text{phase}}$$

However, the -Y| configuration is not the only valid one for connecting three-phase voltage source or load elements together.

Delta Connection

The delta connection has three wires, and there is a no neutral point. The delta connection is shown in the figure below. The line voltage of the delta connection is equal to the phase voltage.



Kirchhoff's Voltage Law in Delta Connections

One quick check of this is to use Kirchhoff's Voltage Law to see if the three voltages around the loop add up to zero. If they do, then there will be no voltage available to push current around and around that loop, and consequently, there will be no circulating current.

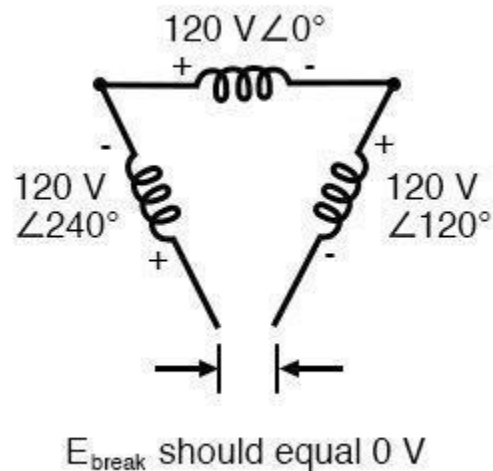
Starting with the top winding and progressing counter-clockwise, our KVL expression looks something like this:

$$(120 \text{ V} \angle 0^\circ) + (120 \text{ V} \angle 240^\circ) + (120 \text{ V} \angle 120^\circ)$$

Does it all equal 0 ?

Yes!

Indeed, if we add these three vector quantities together, they do add up to zero. Another way to verify the fact that these three voltage sources can be connected together in a loop without resulting in circulating currents is to open up the loop at one junction point and calculate the voltage across the break: (figure below)



The voltage across open Δ should be zero.

Starting with the right winding ($120 \text{ V} \angle 120^\circ$) and progressing counter-clockwise, our KVL equation looks like this:

$$(120 \text{ V} \angle 120^\circ) + (120 \text{ V} \angle 0^\circ) + (120 \text{ V} \angle 240^\circ) + E_{\text{break}} = 0$$

$$0 + E_{\text{break}} = 0$$

$$E_{\text{break}} = 0$$

Sure enough, there will be zero voltage across the break, telling us that no current will circulate within the triangular loop of windings when that connection is made complete.

Having established that a Δ -connected three-phase voltage source will not burn itself to a crisp due to circulating currents, we turn to its practical use as a source of power in three-phase circuits.

Because each pair of line conductors is connected directly across a single winding in a Δ circuit, the line voltage will be equal to the phase voltage.

Conversely, because each line conductor attaches at a node between two windings, the line current will be the vector sum of the two joining phase currents.

Not surprisingly, the resulting equations for a Δ configuration are as follows:

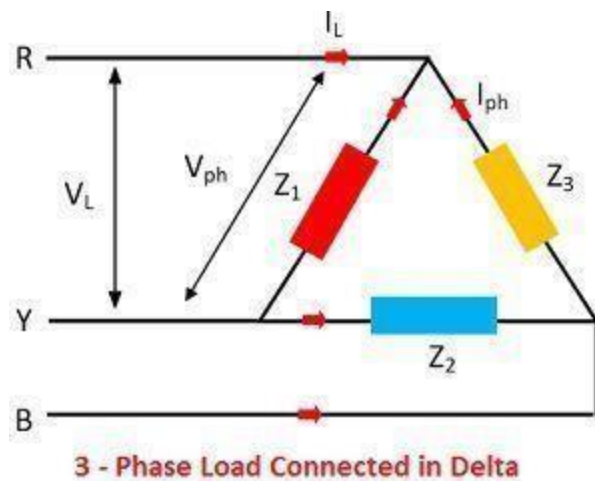
For Δ ("delta") circuits:

$$E_{\text{line}} = E_{\text{phase}}$$

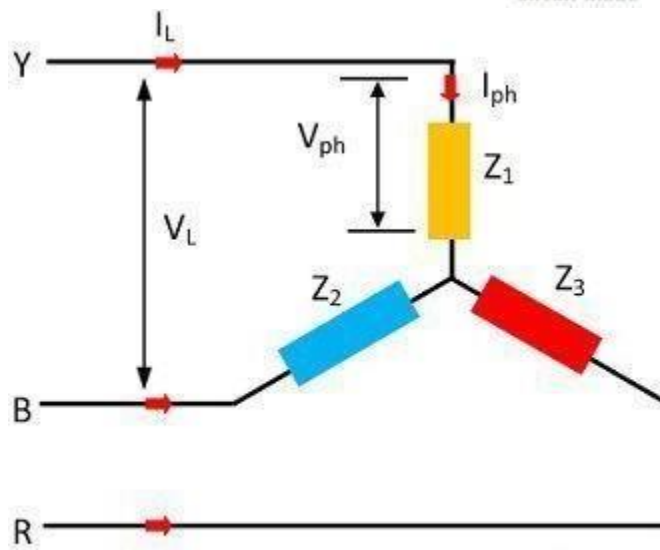
$$I_{\text{line}} = \sqrt{3} I_{\text{phase}}$$

Connection of Loads in Three Phase System

The loads in the three-phase system may also connect in the star or delta. The three phase loads connected in the delta and star is shown in the figure below.



Circuit Globe

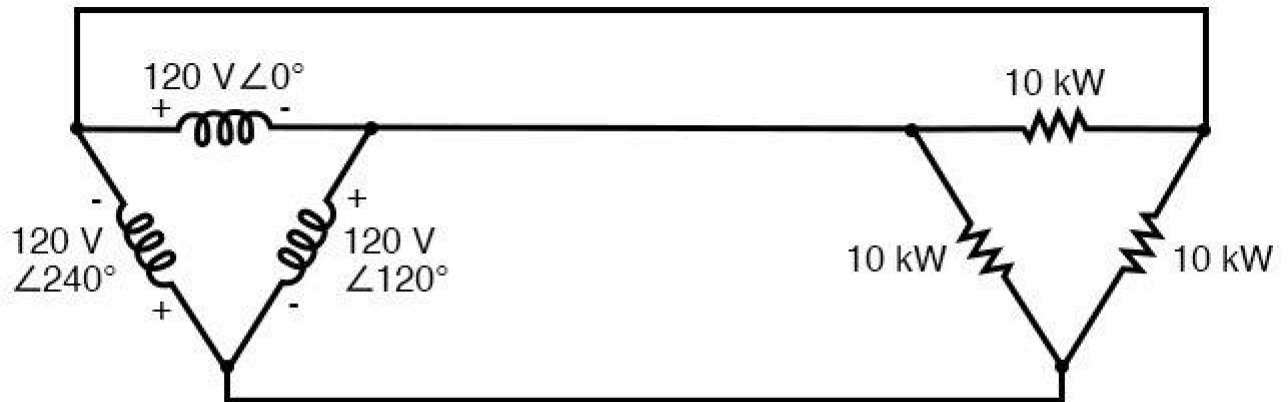


Circuit Globe

The three phase load may be balanced or unbalanced. If the three loads (impedances) Z_1 , Z_2 and Z_3 has the same magnitude and phase angle then the three phase load is said to be a balanced load. Under balance condition, all the phases and the line voltages are equal in magnitude.

Delta Connection Example Circuit Analysis

Let's see how this works in an example circuit: (Figure below)



The load on the Δ source is wired in a Δ .

The load on the Δ source is wired in a Δ .

With each load resistance receiving 120 volts from its respective phase winding at the source, the current in each phase of this circuit will be 83.33 amps:

$$I = \frac{P}{E}$$

$$I = \frac{10 \text{ kW}}{120 \text{ V}}$$

$$I = 83.33 \text{ A (for each load resistor and source winding)}$$

$$I_{\text{line}} = \sqrt{3} I_{\text{phase}}$$

$$I_{\text{line}} = \sqrt{3} (83.33 \text{ A})$$

$$I_{\text{line}} = 144.34 \text{ A}$$

Advantages of the Delta Three-Phase System

So each line current in this three-phase power system is equal to 144.34 amps, which is substantially more than the line currents in the Y-connected system we looked at earlier.

One might wonder if we've lost all the advantages of three-phase power here, given the fact that

we have such greater conductor currents, necessitating thicker, more costly wire.

The answer is no. Although this circuit would require three number 1 gauge copper conductors (at 1000 feet of distance between source and load this equates to a little over 750 pounds of copper for the whole system), it is still less than the 1000+ pounds of copper required for a single-phase system delivering the same power (30 kW) at the same voltage (120 volts conductor-to-conductor).

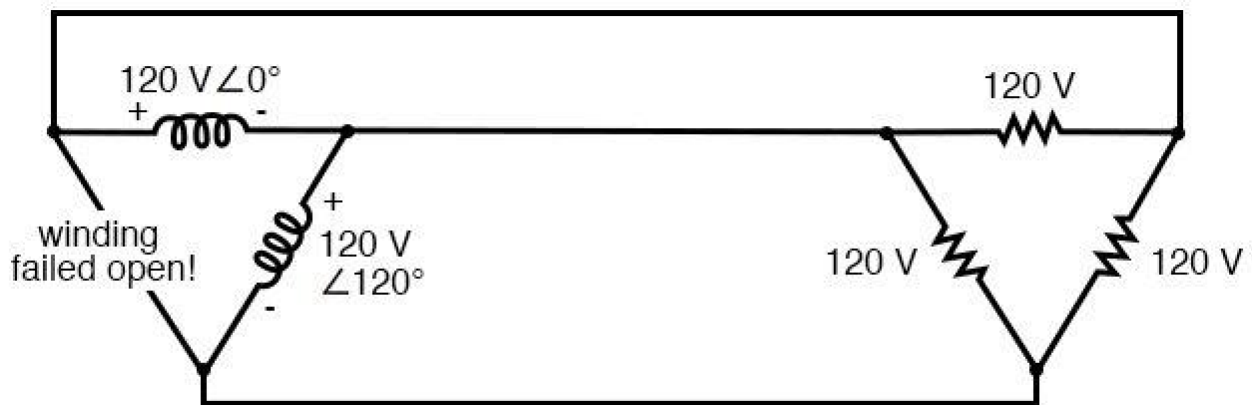
One distinct advantage of a Δ -connected system is its lack of a neutral wire. With a Y-connected system, a neutral wire was needed in case one of the phase loads were to fail open (or be turned off), in order to keep the phase voltages at the load from changing.

This is not necessary (or even possible!) in a Δ -connected circuit.

With each load phase element directly connected across a respective source phase winding, the phase voltage will be constant regardless of open failures in the load elements.

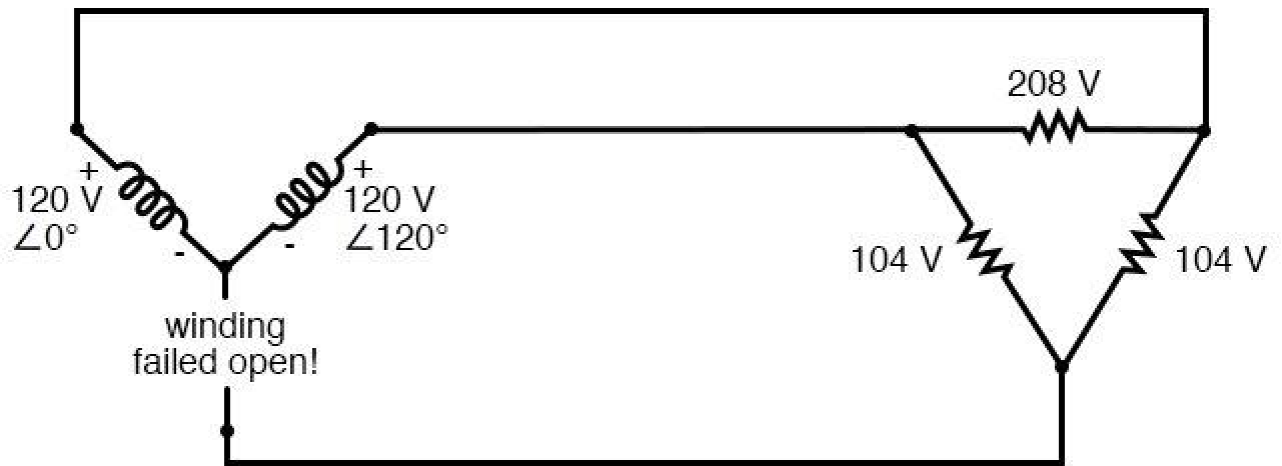
Perhaps the greatest advantage of the Δ -connected source is its fault tolerance.

It is possible for one of the windings in a Δ -connected three-phase source to fail open (Figure below) without affecting load voltage or current!



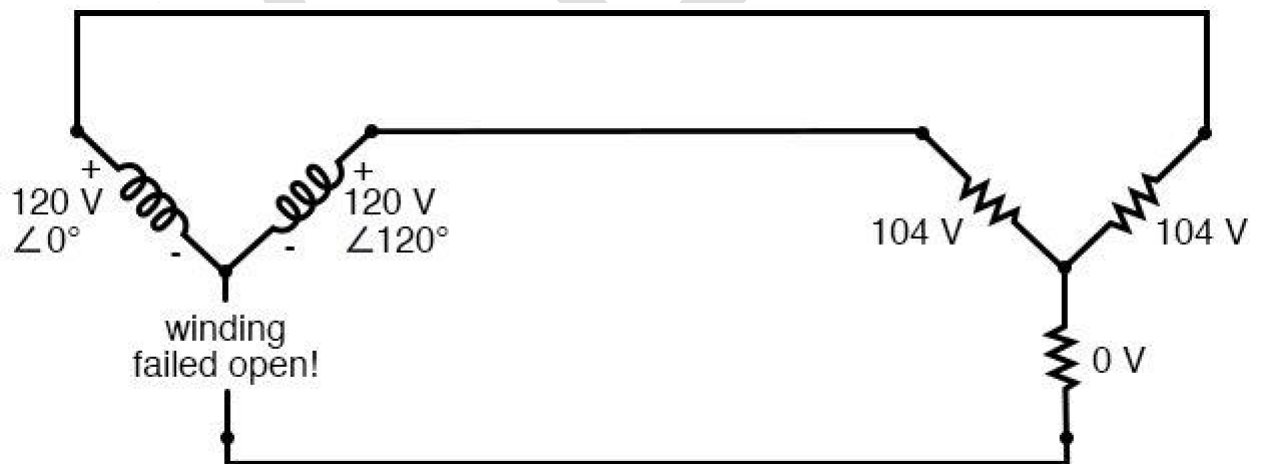
Even with a source winding failure, the line voltage is still 120 V, and the load phase voltage is still 120 V. The only difference is extra current in the remaining functional source windings.

The only consequence of a source winding failing open for a Δ -connected source is increased phase current in the remaining windings. Compare this fault tolerance with a Y-connected system suffering an open source winding in the figure below.



Open “Y” source winding halves the voltage on two loads of a Δ connected the load.

With a Δ -connected load, two of the resistances suffer reduced voltage while one remains at the original line voltage, 208. A Y-connected load suffers an even worse fate (Figure below) with the same winding failure in a Y-connected source.



Open source winding of a “Y-Y” system halves the voltage on two loads and loses one load entirely.

In this case, two load resistances suffer reduced voltage while the third loses supply voltage completely! For this reason, Δ -connected sources are preferred for reliability.

However, if dual voltages are needed (e.g. 120/208) or preferred for lower line currents, Y-connected systems are the configuration of choice.

REVIEW:

- The conductors connected to the three points of a three-phase source or load are called *lines*.
- The three components comprising a three-phase source or load are called *phases*.
- *Line voltage* is the voltage measured between any two lines in a three-phase circuit.
- *Phase voltage* is the voltage measured across a single component in a three-phase source or load.
- *Line current* is the current through any one line between a three-phase source and load.
- *Phase current* is the current through any one component comprising a three-phase source or load.
- In balanced Y circuits, the line voltage is equal to phase voltage times the square root of 3, while the line current is equal to phase current.

For "Y" circuits:

$$E_{\text{line}} = \sqrt{3} E_{\text{phase}}$$

$$I_{\text{line}} = I_{\text{phase}}$$

- In balanced Δ circuits, the line voltage is equal to phase voltage, while the line current is equal to phase current times the square root of 3.

For Δ ("delta") circuits:

$$E_{\text{line}} = E_{\text{phase}}$$

$$I_{\text{line}} = \sqrt{3} I_{\text{phase}}$$

- Δ -connected three-phase voltage sources give greater reliability in the event of winding failure than Y-connected sources. However, Y-connected sources can deliver the same amount of power with less line current than Δ -connected sources.

THREE PHASE POWER MEASUREMENT USING WATTMETER

Various methods are used for **measurement of three phase power** in three phase circuits on the basis of number of wattmeters used. We have three methods to discuss:

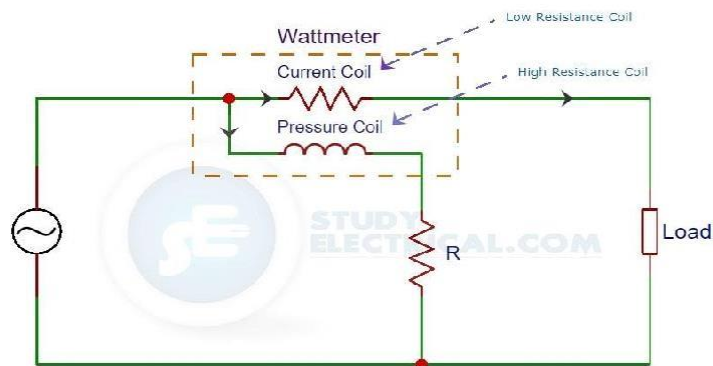
1. Three wattmeters method
2. Two wattmeters method
3. Single wattmeter method.

What is a Wattmeter?

A wattmeter is an equipment used to measure power in a circuit. It consists of two types of coils. They are:

- A **Current Coil** that possesses a *low resistance*.
- A **Pressure or Potential Coil** that possesses a *high resistance*.

The current coil is connected in series with the line carrying current. The pressure coil is connected across the two points whose potential difference is to be measured. Refer the figure for the connections of a wattmeter.



Connection of a Wattmeter

A wattmeter shows a reading which proportional to the product of three values. They are

- Current (I) through its current coil.
- Potential difference (V) across its pressure coil.
- Cosine of the angle between voltage and current ($\cos\phi$).

$$P = VI\cos(\phi)$$

A comparison between the methods of measuring power in a three-phase circuit is shown in the table below.

Three Wattmeter Method	Used for measurement of 3 phase, 4 wire circuits. Both balanced and unbalanced loads.
One Wattmeter Method	Used in Balanced 3 phase, 3 wire load circuit.
Two Wattmeter Method	Used in both balanced and unbalanced 3 phase, 3 wire circuits

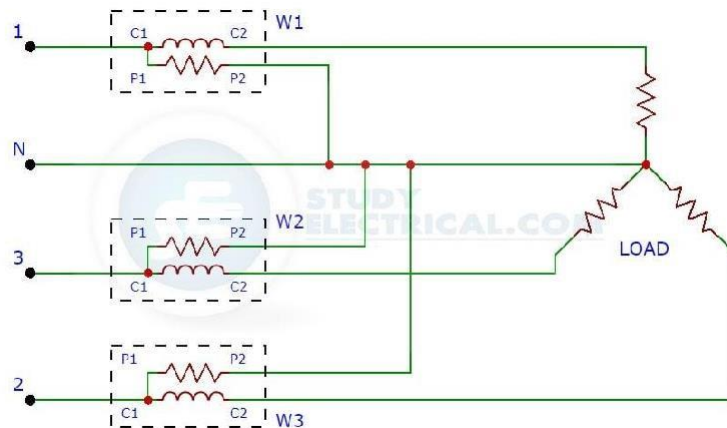
Let us discuss one by one each method in detail.

Three Wattmeter Method

Now we will explain the measurement three-phase power measurement using three wattmeter method.

Three wattmeter method is used to measure power in 3 phase, 4-wire circuits. However, this method can also be used in 3 phase, 3 wire delta connected load, where power consumed by each load is required to be determined separately.

The figure below shows the three wattmeter connection of 3 phase, 4 wire star connected load.



Three Wattmeter Method

As indicated in the figure, the three wattmeters are connected in each of the three phases to measure three-phase power usage of the load whether star or delta connected.

The current coil of each wattmeter carries the current of one phase only and the pressure coil measures the phase voltage of the phase. Hence, each wattmeter measures the power in a single phase. The total power in the load is given by the algebraic sum of the readings of the three wattmeters.

$$P = W_1 + W_2 + W_3$$

where , $W_1 = V_1 \cdot I_1$, $W_2 = V_2 \cdot I_2$, $W_3 = V_3 \cdot I_3$

Disadvantages of Three Wattmeter Method

While using three wattmeter method following difficulty is met with:

- In the case of 3 phase, 3 wire star connected load, it is difficult to get a neutral point which is required for connection. In special cases, when this method is necessary to use, an 'artificial star' can be formed.
- In case of delta connected circuits, the difficulty in using this method is due to fact that the phase coils are required to be broken for inserting current coils of wattmeters.

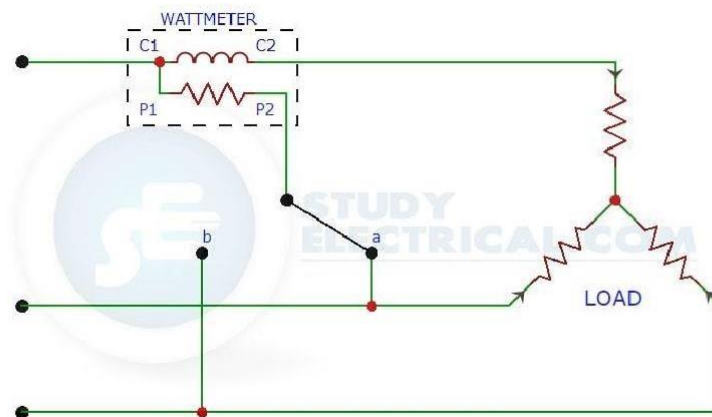
To measure power it is not necessary to use three wattmeter, even two wattmeters can be used for the purpose.

Except for 3 phase, 4 wire unbalanced load, three-phase power are measured using only Two Wattmeter Method.

One Wattmeter Method

The next method we are going to discuss is the one wattmeter method.

In this method of three-phase power measurement, the current coil is connected in any one line and the pressure coil is connected alternatively between this and the other two lines. The connection diagram is shown in the figure below.



One Wattmeter Method

So we will get two readings for a balanced load. The two readings so obtained, correspond to those obtained by the normal two wattmeter method.

A balanced load is a load that draws the same current from each phase of the three-phase system, while an unbalanced load has at least one of those currents different from the rest.

In balanced 3-wire, 3-phase load circuit the power in each phase is equal. Therefore, the total power of the circuit can be determined by multiplying the power measured in any one phase by three.

$$\begin{aligned} \text{Total power in balanced load} &= 3 \times \text{Power per Phase} \\ &= 3 \times \text{Wattmeter reading} \end{aligned}$$

Disadvantages of One Wattmeter Method

This method is not of as much universal application as the two wattmeter method because it is restricted to fairly balance loads only. Even a slight degree of unbalance in the loading produce a large error in the measurement.

However, it may be conveniently applied, for instance, when it is desired to find the power input to a factory motor in order to check the load up on the motor.

Two Wattmeter Method

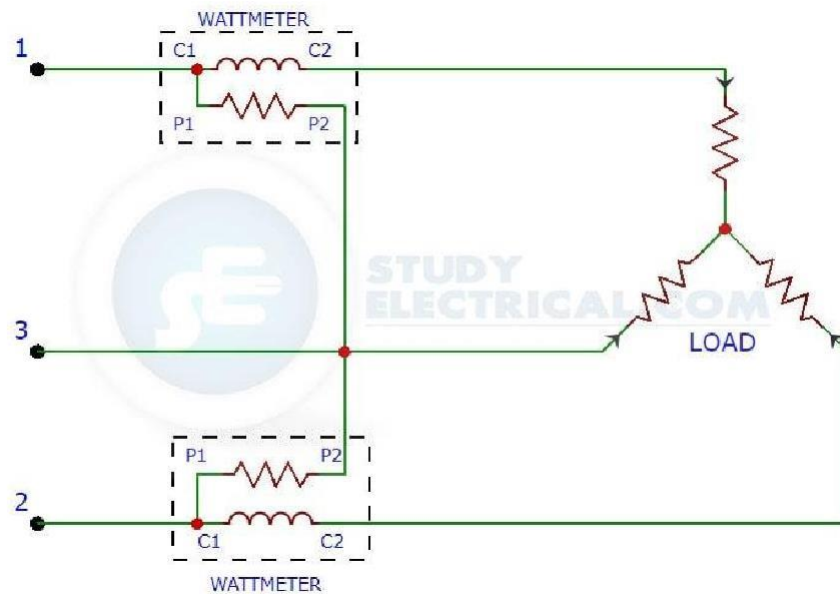
As the name indicates, in this method two wattmeters are used to measure three-phase power. This is the most popular method among the three.

This method is generally used for the measurement of power in 3 phase, 3-wire load circuits. It

can be used to measure power in star/delta connected load in balanced or unbalanced condition.

Remember a balanced load is a load that draws the same current from each phase of the three-phase system, while an unbalanced load has at least one of those currents different from the rest.

In two wattmeter method, the current coils of the two wattmeters are inserted in any two lines and pressure coil of each wattmeter is joined to the third line. Refer the figure below for better understanding.



Two Wattmeter Method

The figure above shows the two wattmeter connection of star connected load. Similarly, delta connected loads are also used. Two wattmeter method can be used irrespective of balanced or unbalanced load.

The algebraic sum of two wattmeter reading gives the total power in the 3-phase, 3 wire star-connected or delta connected load circuits whether the load is balanced or unbalanced.

$$P = W1 + W2$$

THREE PHASE BALANCED SYSTEMS PROBLEMS

1. The input power to a 3-phase a.c. motor is measured as 5kW. If the voltage and current to the motor are 400V and 8.6A respectively, determine the power factor of the system?

$$\text{Power } P=5000\text{W,}$$

$$\text{line voltage } V_L = 400 \text{ V,}$$

$$\text{line current, } I_L = 8.6\text{A and}$$

$$\text{power, } P = \sqrt{3} V_L I_L \cos \phi$$

Hence

$$\text{power factor} = \cos \phi = \frac{P}{\sqrt{3} V_L I_L}$$

$$= \frac{5000}{\sqrt{3} (400) (8.6)}$$

$$= \mathbf{0.839}$$

2. Two wattmeters are connected to measure the input power to a balanced 3-phase load by the two-wattmeter method. If the instrument readings are 8kW and 4kW, determine (a) the total power input and (b) the load power factor.

(a) Total input power,

$$P = P_1 + P_2 = 8 + 4 = \mathbf{12\text{kW}}$$

$$(b) \tan \phi = \frac{\sqrt{3}(P_1 - P_2)}{(P_1 + P_2)}$$

$$= \frac{\sqrt{3} (8 - 4)}{(8 + 4)}$$

$$= \frac{\sqrt{3} (4)}{12}$$

$$= \frac{\sqrt{3}}{3}$$

$$= \frac{1}{\sqrt{3}}$$

$$\text{Hence } \phi = \tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$$

$$\text{Power factor} = \cos \phi = \cos 30^\circ = \mathbf{0.866}$$

3. Two wattmeters connected to a 3-phase motor indicate the total power input to be 12kW. The power factor is 0.6. Determine the readings of each wattmeter.

If the two wattmeters indicate P_1 and P_2 respectively

$$\text{Then } P_1 + P_2 = 12\text{kW} \text{ ---(1)}$$

$$\tan \phi = \sqrt{3}(P_1 - P_2)/(P_1 + P_2)$$

And power factor = 0.6 = $\cos \phi$.

Angle $\phi = \cos^{-1} 0.6 = 53.13^\circ$ and

$$\tan 53.13^\circ = 1.3333.$$

Hence

$$1.3333 = \sqrt{3}(P_1 - P_2)/12$$

From which,

$$P_1 - P_2 = 12(1.3333) / \sqrt{3}$$

$$\text{i.e. } P_1 - P_2 = 9.237\text{kW} \text{----- (2)}$$

Adding Equations (1) and (2) gives:

$$2P_1 = 21.237$$

$$\text{i.e. } P_1 = 21.237/2$$

= 10.62kW Hence wattmeter 1 reads 10.62kW From Equation (1), wattmeter 2 reads

$$(12 - 10.62) = 1.38\text{kW}$$

4. Three loads, each of resistance 30, are connected in star to a 415 V, 3 -phase supply. Determine

(a) the system phase voltage, (b) the phase current and (c) the line current.

A '415 V, 3-phase supply' means that 415 V is the line voltage, V_L

(a) For a star connection, $V_L = \sqrt{3}V_p$ Hence phase voltage, $V_p = V_L/\sqrt{3}$

$$= 415 / \sqrt{3}$$

$$= \mathbf{239.6 \text{ V or } 240 \text{ V}}$$

correct to 3 significant figures

(b) Phase current, $I_p = V_p/R_p$

$$= 240/30$$

$$= 8 \text{ A}$$

(c) For a star connection, $I_p = I_L$. Hence the line current, $I_L = 8 \text{ A}$

5. Three identical coils, each of resistance 10ohm and inductance 42mH are connected (a) in star and (b) in delta to a 415V , 50 Hz , 3-phase supply. Determine the total power dissipated in each case.

(a) **Star connection**

Inductive reactance,

$$X_L = 2\pi f L = 2\pi (50) (42 \times 10^{-3}) = 13.19$$

Phase impedance,

$$Z_p = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{(10)^2 + (13.19)^2} = 16.55$$

Line voltage, $V_L = 415 \text{ V}$

And phase voltage,

$$V_p = V_L / \sqrt{3} = 415 / \sqrt{3} = 240 \text{ V.}$$

Phase current,

$$I_p = V_p / Z_p = 240 / 16.55 = 14.50 \text{ A. Line current,}$$

$$I_L = I_p = 14.50 \text{ A.}$$

Power factor = $\cos \phi = R_p / Z_p = 10 / 16.55 = 0.6042$ lagging.

Power dissipated,

$$P = \sqrt{3} V_L I_L \cos \phi = \sqrt{3} (415) (14.50) (0.6042) = \mathbf{6.3\text{kW}}$$
 (Alternatively,

$$P = 3 I^2 R = 3 (14.50)^2 (10) = \mathbf{6.3\text{kW}})$$

(b) **Delta connection**

$$V_L = V_p = 415 \text{ V,}$$

$Z_p = 16.55$, $\cos \phi = 0.6042$ lagging (from above). Phase current,

$I_p = V_p/Z_p = 415/16.55 = 25.08\text{A}$. Line current,

$$I_L = \sqrt{3}I_p = \sqrt{3}(25.08) = 43.44\text{A}.$$

Power dissipated,

$$P = \sqrt{3} V_L I_L \cos \phi$$

$$= \sqrt{3} (415)(43.44)(0.6042) = \mathbf{18.87\text{kW}}$$

(Alternatively,

$$P = 3I^2R$$

$$= 3(25.08)^2(10) = \mathbf{18.87\text{ kW}}$$

6. A 415V, 3-phase a.c. motor has a power output of 12.75kW and operates at a power factor of 0.77 lagging and with an efficiency of 85 per cent. If the motor is delta-connected, determine (a) the power input, (b) the line current and (c) the phase current.

(a) Efficiency = power output/power input.

Hence

$$(85/100) = 12.750 \text{ power input from which, Power input} = 12.750 \times 100/85$$

$$= \mathbf{15\,000\text{W}} \text{ or } \mathbf{15\text{Kw}}$$

(b) Power, $P = \sqrt{3} V_L I_L \cos \phi$, hence

(c) **line current,**

$$I_L = P / \sqrt{3} (415) (0.77)$$

$$= 15\,000 / \sqrt{3} (415) (0.77)$$

$$= \mathbf{27.10\text{A}}$$

(d) For a delta connection, $I_L = \sqrt{3} I_p$,

Hence

$$\text{Phase current, } I_p = I_L / \sqrt{3}$$

$$= 27.10 / \sqrt{3}$$

$$= 15.65A$$

7. A 400V, 3-phase star connected alternator supplies a delta-connected load, each phase of which has a resistance of $30\ \Omega$ and inductive reactance $40\ \Omega$. Calculate (a) the current supplied by the alternator and (b) the output power and the kVA of the alternator, neglecting losses in the line between the alternator and load.

A circuit diagram of the alternator and load is shown in Fig.

(a) Considering the load:

$$\text{Phase current, } I_p = V_p / Z_p$$

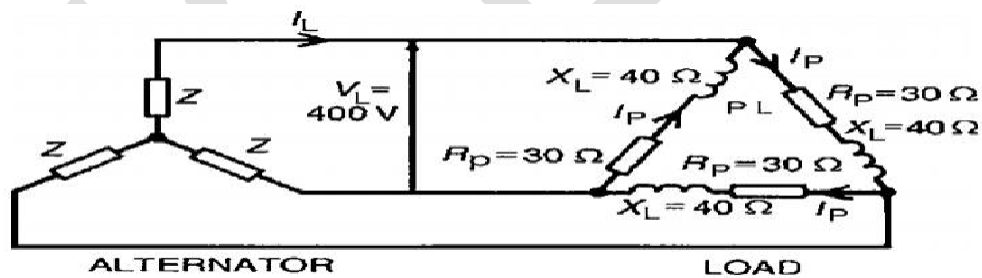
$V_p = V_L$ for a delta connection,

$$\text{Hence } V_p = 400V.$$

Phase impedance,

$$Z_p = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{(30^2 + 40^2)} = 50$$



Figure

$$\text{Hence } I_p = V_p / Z_p = 400 / 50 = 8A.$$

For a delta-connection,

$$\text{Line current, } I_L = \sqrt{3} I_p = \sqrt{3} (8) = 13.86 A.$$

Hence **13.86A** is the current supplied by the alternator.

(b) Alternator output power is equal to the power Dissipated by the load

I.e. $P = \sqrt{3} VLIL \cos \phi$, Where $\cos \phi = R_p/Z_p = 30/50 = 0.6$.

Hence $P = \sqrt{3} (400) (13.86) (0.6) = \mathbf{5.76kW}$.

Alternator output kVA,

$S = \sqrt{3} VLIL = \sqrt{3} (400) (13.86)$

9.60 kVA.

APPENDIX I
CONTENT BEYOND THE SYLLABUS

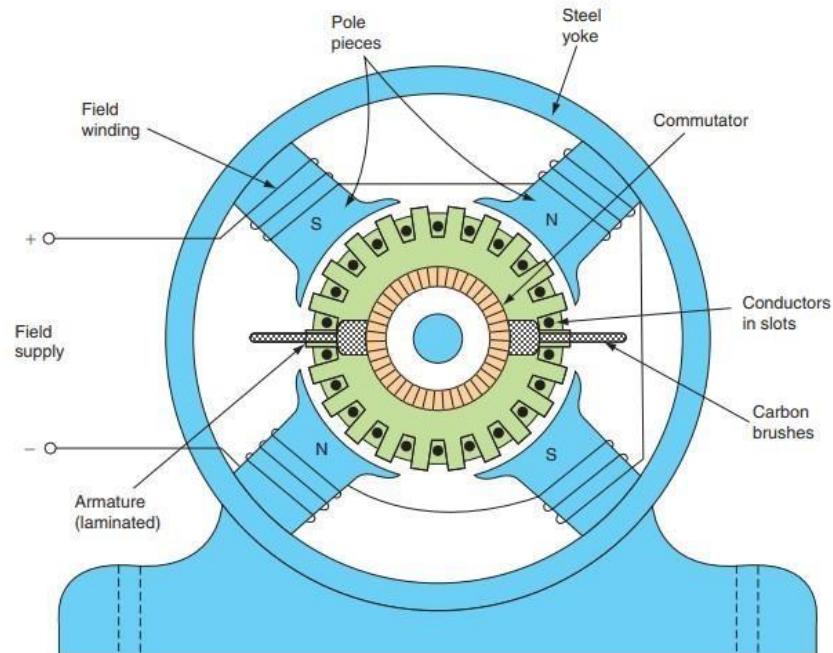
APPENDIX I
CONTENT BEYOND THE SYLLABUS
DC MACHINES

INTRODUCTION TO DC MACHINES

- **DC Machines** are types of electrical machines that use dc current in the case of dc motors and generates dc voltages in case of dc generator.
- DC motor transforms electrical power into mechanical power and the generator converts mechanical power into electrical
- As in power, generation system, and industries mostly ac machines like an induction motor, synchronous motor, generators are employed but the use of dc machines cannot be denied due to its constant speed providing the capability.
- The physical construction and designing of both dc motors and generators are alike to each other. The dc generator is used in some safe environment where is no need of special protection and circuitry for the generator.
- While the motor is employed in such an environment where it can be easily affected by environmental conditions like moisture, dust, so it needs a special structure that can provide resistant to dust, fire, and some other related effects.
- As we are familiar with the common use of dc battery that used only for some limited applications where less amount of energy is required but such applications where a large amount of dc power is needed dc machines is the best replacement of the battery.

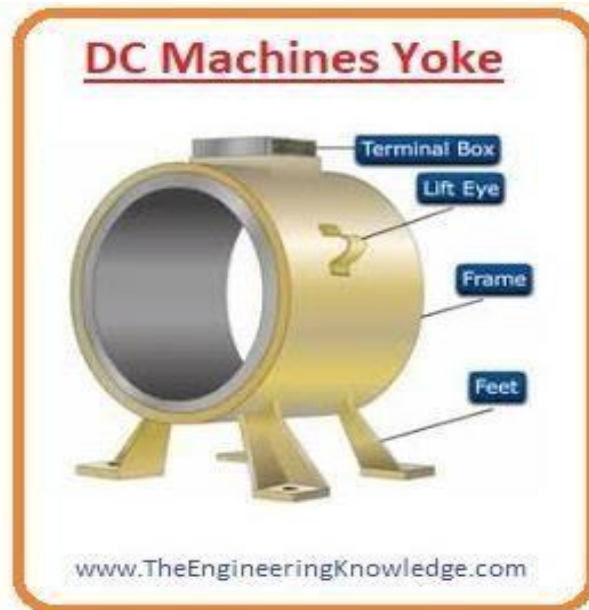
DC MACHINES CONSTRUCTION

- There are numerous parts are exists in the designing and structure of dc machines like rotor, stator, windings and some others, all these part are described here with the detailed.



DC Machines Yoke

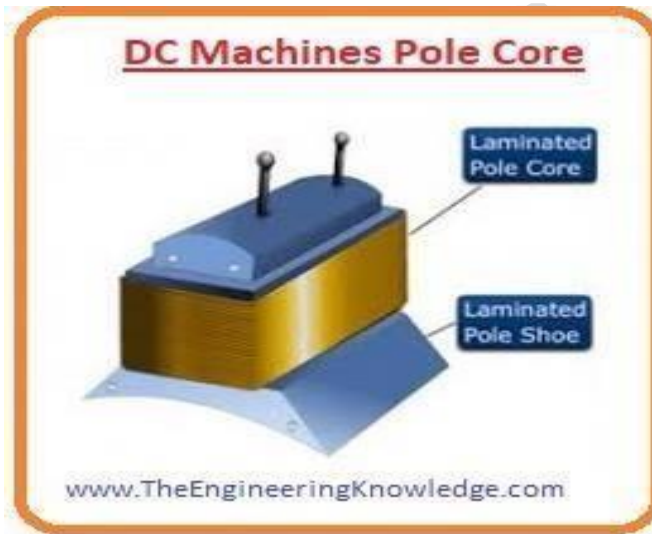
- The yoke of machines is also known as the frame, the main working of this part is to provide protection to the internal circuitry of a machine from the outer environment, temperature, moisture, and some other factors.
- This part of the machine is constructed with the cast steel and cast iron.



DC Machines Pole Core

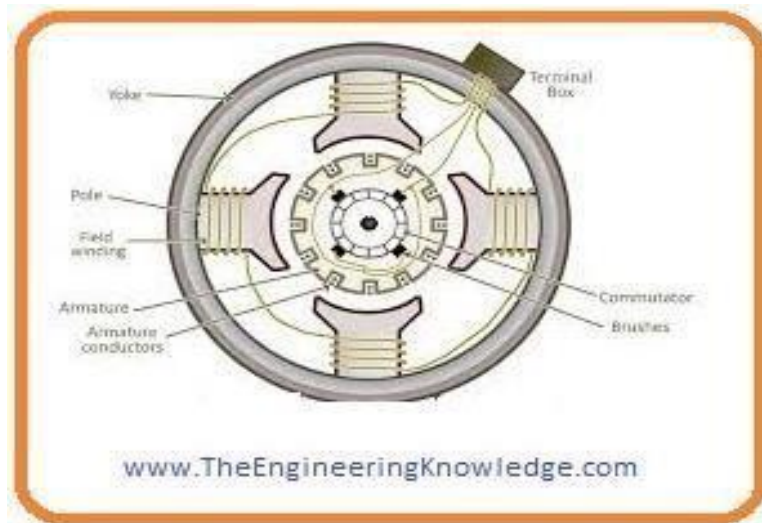
- At stator of the dc machines, the poles are of an electromagnet the windings on these poles are known as the field windings.
- The input provided at the stator connected with the field windings generate flux at the stator and make poles electromagnet.

These poles are constructed with cast steel, cast iron.



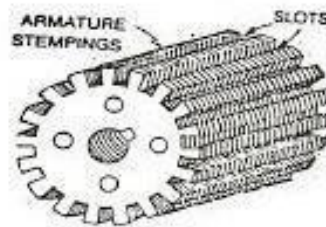
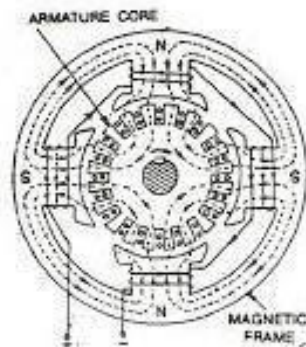
DC Machines Field Windings

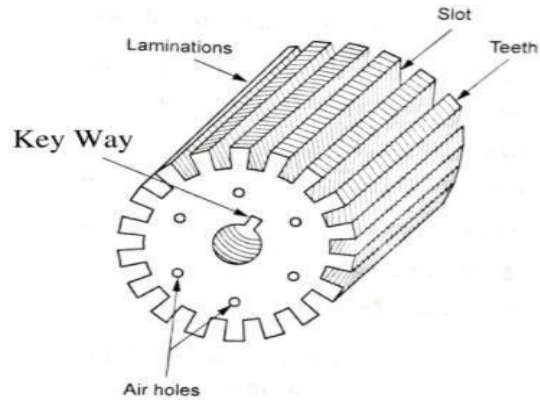
- These windings are wound at the stator part of machines on the poles at the stator. These windings are constructed with copper. The current provided at these windings generates flux and makes poles electromagnet.



DC Machines Armature Core

- The core of armature consists of a large no of slots and armature windings are located in these slots.
- || It has less reluctance path for the interaction of stator flux with the armature windings, this core is constructed with the less reluctance material like cast iron.
- || And there are laminations of different substance is used to reduce the eddy current losses.





DC machines Armature Windings

- The windings wound on the rotor of the dc machine is known as the armature windings. When the rotor rotates due to flux linking of stator the voltage induced in this part of machines.
- These windings are constructed with copper like the stator windings.

DC Machines Commutator

- The commutator is slip rings mounted at the shaft of the machines the main purpose of these commutators is to transmit current from the armature windings to the load.
- These commutators also convert ac power generated by the machine into the dc power, we will discuss this phenomenon how ac converts into dc by these commutators in coming tutorials.

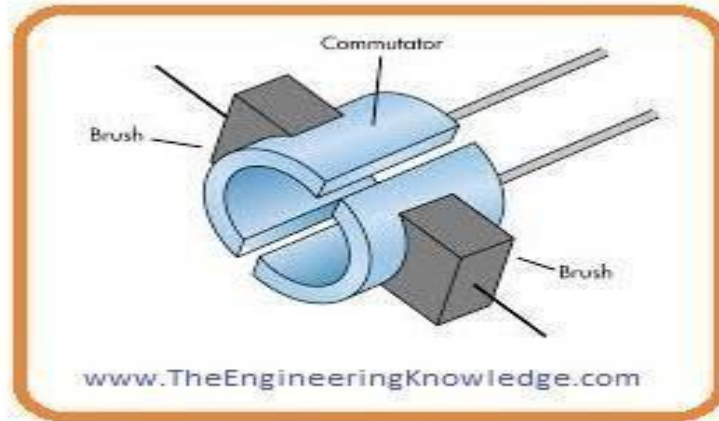


Carbon Brushes

- These brushes are connected with the commutators and get current from the commutator

and provides to the load.

- These are constructed with the carbon and their main function is to reduce the sparking at load and machine connection points.

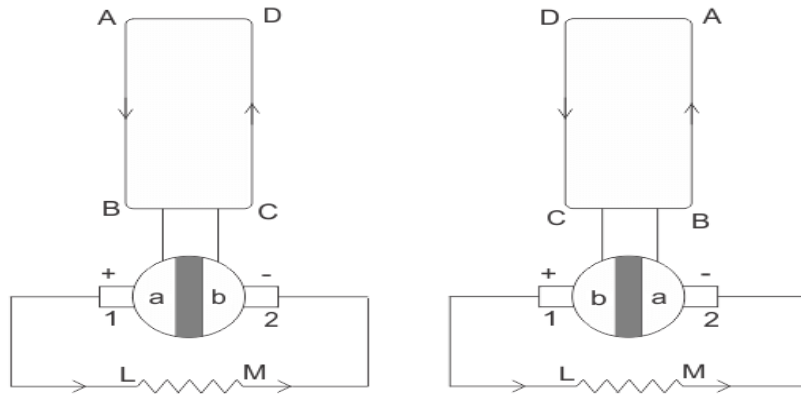


DC GENERATORS

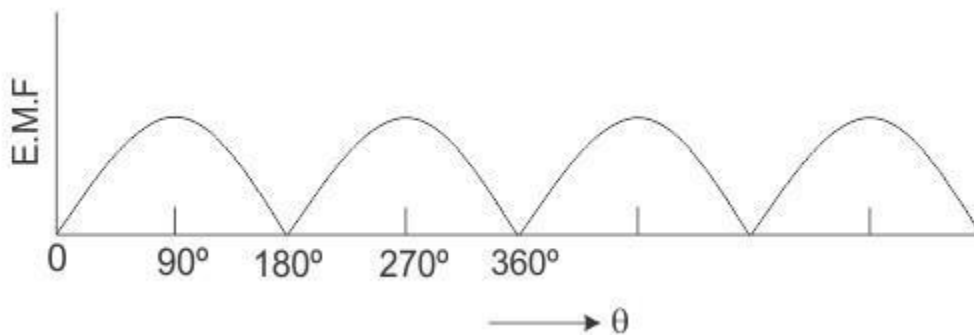
What is a DC Generator?

A DC generator is an electrical machine whose main function is to convert mechanical energy into electricity. When conductor slashes magnetic flux, an emf will be generated based on the electromagnetic induction principle of Faraday's Laws. This electromotive force can cause a flow of current when the conductor circuit is closed.

Working Principle of DC Generator



We can see that in the first half of the revolution current always flows along ABLMCD, i.e., brush no 1 in contact with segment a. In the next half revolution, in the figure, the direction of the induced current in the coil is reversed. But at the same time the position of the segments a and b are also reversed which results that brush no 1 comes in touch with the segment b. Hence, the current in the load resistance again flows from L to M. The waveform of the current through the load circuit is as shown in the figure. This current is unidirectional.



The above content is the basic **working principle of DC generator**, explained by single loop generator model. The positions of the brushes of DC generator are so that the change over of the segments a and b from one brush to other takes place when the plane of rotating coil is at a right angle to the plane of the lines of force. It is to become in that position, the induced EMF in the coil is zero.

EMF EQUATION OF A DC GENERATOR/MOTOR

As the armature rotates, a voltage is generated in its coils. In the case of a generator, the emf of rotation is called the Generated emf or Armature emf and is denoted as $E_r = E_g$. In the case of a motor, the emf of rotation is known as Back emf or Counter emf and represented as $E_r = E_b$.

The expression for emf is same for both the operations, i.e., for Generator as well as for Motor.

Derivation of EMF Equation of a DC Machine – Generator and Motor

Let,

- **P** – number of poles of the machine
- ϕ – Flux per pole in Weber.
- **Z** – Total number of armature conductors.
- **N** – Speed of armature in revolution per minute (r.p.m).
- **A** – number of parallel paths in the armature winding.

In one revolution of the armature, the flux cut by one conductor is given as:

$$\text{Flux cut by one conductor} = P\phi \text{ wb} \dots \dots (1)$$

Time taken to complete one revolution is given as:

$$t = \frac{60}{N} \text{ seconds} \dots \dots (2)$$

Therefore, the average induced e.m.f in one conductor will be:

$$e = \frac{P\phi}{t} \dots \dots (3)$$

Putting the value of (t) from Equation (2) in the equation (3) we will get

$$e = \frac{P\phi}{60/N} = \frac{P\phi N}{60} \text{ volts} \dots \dots (4)$$

The number of conductors connected in series in each parallel path = Z/A .

Therefore, the average induced e.m.f across each parallel path or the armature terminals is given by the equation shown below:

$$E = \frac{P\phi N}{60} \times \frac{Z}{A} = \frac{PZ\phi N}{60 A} \text{ volts or}$$

$$E = \frac{PZ\phi n}{A} \dots\dots (5)$$

Where n is the speed in revolution per second (r.p.s) and given as:

$$n = \frac{N}{60}$$

For a given machine, the number of poles and the number of conductors per parallel path (Z/A) are constant. Hence, equation (5) can be written as:

$$E = K\phi n$$

Where K is a constant and given as:

$$K = \frac{PZ}{A}$$

Therefore, the average induced emf equation can also be written as:

$$E \propto \phi n \text{ or}$$

$$E = K_1 \phi N$$

Where K_1 is another constant and hence induced emf equation can be written as:

$$E \propto \phi N \text{ or}$$

$$E \propto \phi \omega$$

Where ω is the angular velocity in radians/second is represented as:

$$\omega = \frac{2\pi N}{60}$$

Thus, it is clear that the induced emf is directly proportional to the speed and flux per pole. The polarity of induced emf depends upon the direction of the magnetic field and the direction of

rotation. If either of the two is reversed the polarity changes, but if two are reversed the polarity remains unchanged.

This induced emf is a fundamental phenomenon for all the DC Machines whether they are working as a generator or motor.

If the DC Machine is working as a Generator, the induced emf is given by the equation shown below:

$$E_g = \frac{PZ \phi N}{60 A} \text{ volts}$$

Where E_g is the **Generated Emf**

If the DC Machine is working as a Motor, the induced emf is given by the equation shown below:

$$E_b = \frac{PZ \phi N}{60 A} \text{ volts}$$

In a motor, the induced emf is called **Back Emf (E_b)** because it acts opposite to the supply voltage.

Induced emf of DC generator is

$$e = \phi P \frac{N}{60} \times \frac{Z}{A} \text{ volts}$$

For Simple wave wound generator

Numbers of parallel paths are only 2 = A

Therefore,

Induced emf for wave type of winding generator

$$\frac{\phi P N}{60} \times \frac{Z}{2} = \frac{\phi Z P N}{120} \text{ volts}$$

For Simple lap-wound generator

Here, number of parallel paths is equal to number of conductors in one path
i.e. $P = A$

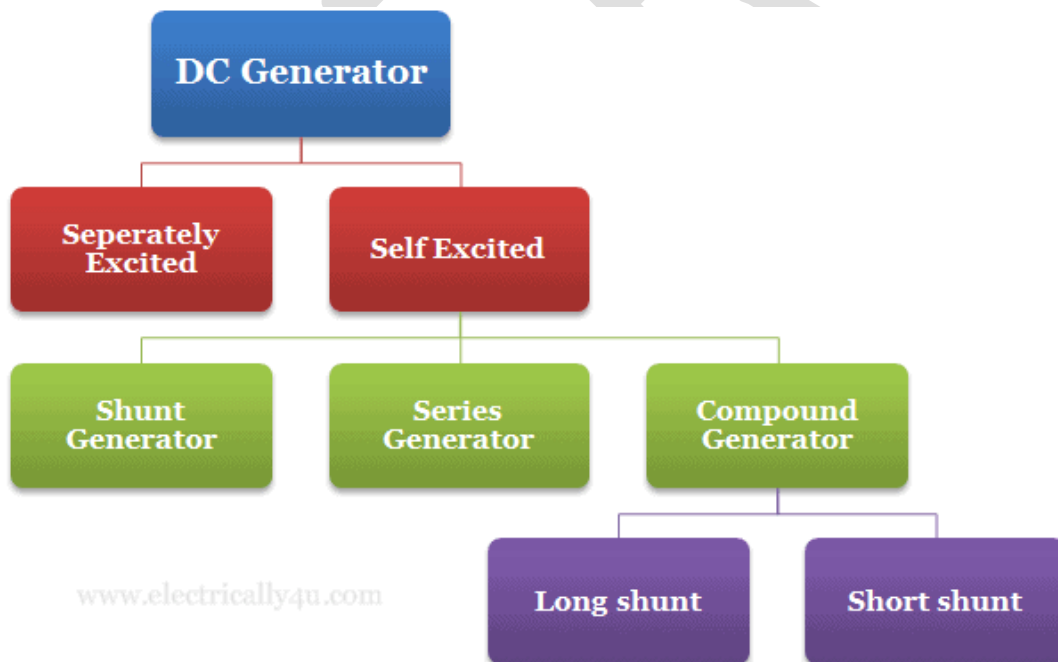
Therefore,

Induced emf for lap-wound generator is

$$E_g = \frac{\phi Z N}{60} \times \frac{P}{A} \text{ volt}$$

TYPES OF DC GENERATOR AND ITS EQUATION

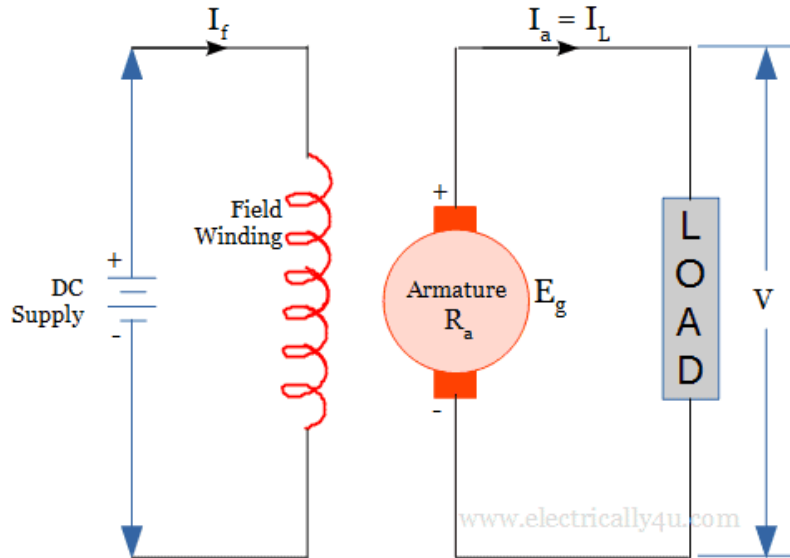
DC generators are classified based on the way in which the field windings are excited. The different types of DC generator are shown below



Before getting into the topic, learn the construction of DC Generator and its working principle.

Separately excited DC Generator

It is a type of DC generator, in which the field windings are excited from a separate source of supply. The following figure shows the circuit diagram of a separately excite dc generator.



In the above circuit diagram,

I_f – Field current, I_a – Armature current, I_L – Load current,

R_a – Armature winding resistance, V – terminal voltage

Let V_{br} be the voltage drop at the brush contacts.

Armature current is given by, $I_a = I_L$

Applying Kirchoff's Voltage Law to the armature circuit, $E_g - I_a R_a - V - V_{br} = 0$

Thus, the generated Emf equation $E_g = I_a R_a + V + V_{br}$

Power developed in the DC generator = $E_g I_a$

Power delivered to the load = $V I_a$

Self- Excited DC Generator

The self-excited generator produces DC output, whose field windings are excited by the current produced by the generator itself. No separate source is used for field excitation.

In this type of generators, some flux is already present in the poles due to residual magnetism. When the armature is rotated with the residual flux, a small emf and hence some current is induced in the armature conductors. This current will produce more flux, which in turn produces more current to flow through the field winding. It will continue until the field current reaches its rated value.

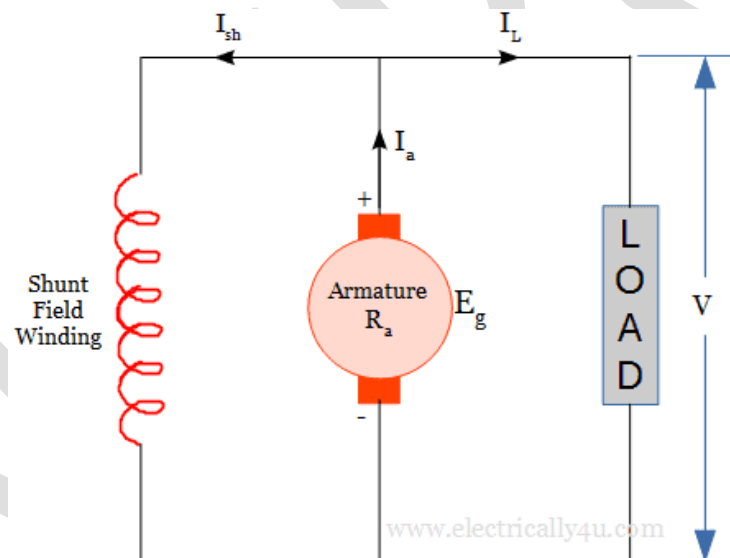
There are three types of self-excited DC generator based on the way, in which the field coils and armature are connected. They are

1. Shunt-wound Generator
2. Series wound Generator
3. Compound wound Generator

DC Shunt Generator

In DC shunt type generator, the field windings are connected across or in parallel with the armature conductors. The field winding has more number of turns and thin wire, having high resistance.

The load is connected across the armature as shown in the diagram below. A small amount of current will flow through the field winding and more current flows through the armature.



In the above circuit diagram,

I_{sh} – Shunt field current, I_a – Armature current, I_L – Load current,

R_a – Armature winding resistance, V – terminal voltage, V_{br} – Brush contact drop

Armature current is given by, $I_a = I_L + I_{sh}$

Shunt field current $I_{sh} = V/R_{sh}$, Where R_{sh} – shunt field resistance

Terminal voltage equation is given by, $V = E_g - I_a R_a - V_{br}$

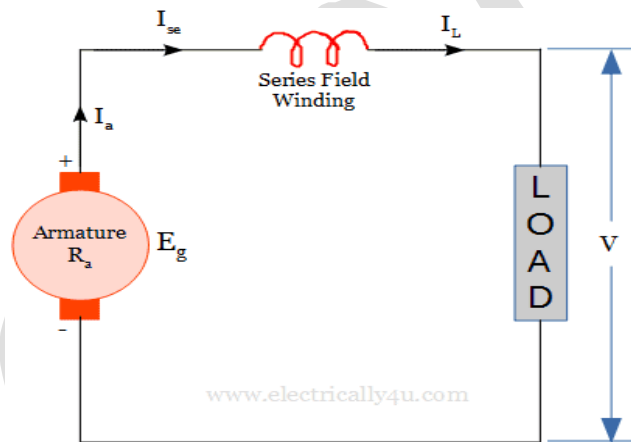
Power developed in the DC generator = $E_g I_a$

Power delivered to the load = $V I_L$

DC Series Generator

As the name says, the field winding is connected in series with the armature conductors. Such generators are called a DC series Generator. They have less number of turns with a thick wire having low resistance.

Here, the load is connected in series with the field winding and armature conductors. So all the current flows through field winding and load.



In the above circuit diagram,

I_{se} – Shunt field current, I_a – Armature current, I_L – Load current,

R_a – Armature winding resistance, V – terminal voltage, V_{br} – Brush contact drop

Armature current is given by, $I_a = I_{se} = I_L$

Terminal voltage equation is given by, $V = E_g - I_a R_a - I_a R_{se} - V_{br}$

Power developed in the DC generator = $E_g I_a$

Power delivered to the load = $V I_L$

DC Compound Generator

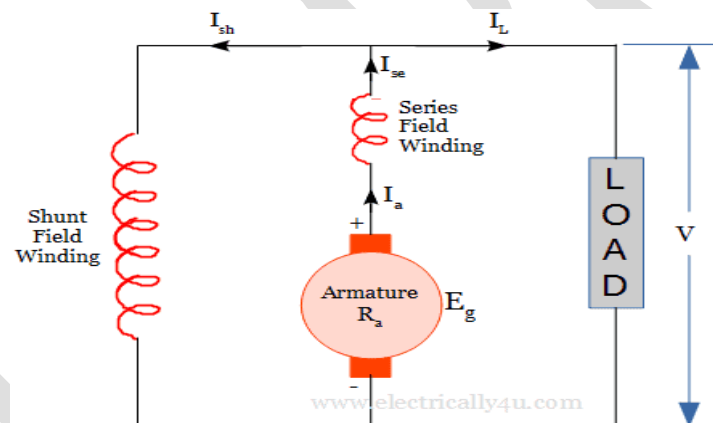
DC compound generator has both shunt field winding and series field winding. One field winding is connected in series with the armature and another field winding is connected in parallel with the armature.

DC Compound generator can be classified into two different types based on the way of connection

1. Long shunt DC Compound generator
2. Short shunt DC Compound generator.

Long shunt DC Compound generator

The below figure shows the circuit diagram of long shunt DC compound generator. In this, shunt field winding is connected in parallel with a combination of series field winding and armature conductors.



In the above circuit diagram,

I_{sh} – Shunt field current, I_{se} – Shunt field current, I_a – Armature current, I_L – Load current,

R_a – Armature winding resistance, V – terminal voltage, V_{br} – Brush contact drop

Armature current is given by, $I_a = I_{se} = I_L + I_{sh}$

Shunt field current $I_{sh} = V/R_{sh}$, Where R_{sh} – shunt field resistance

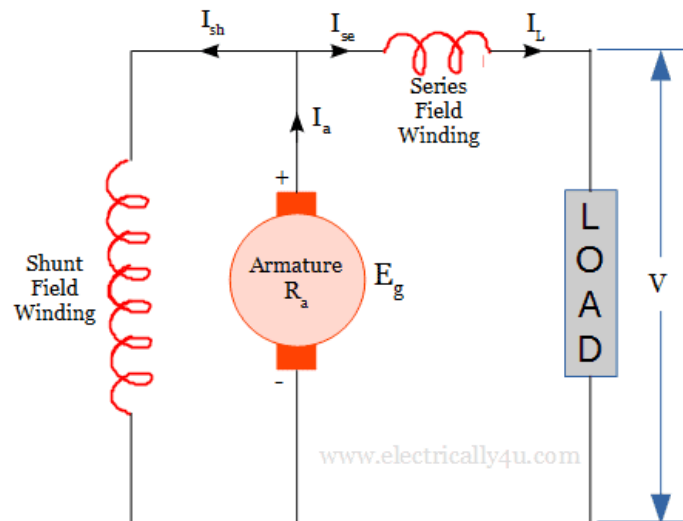
Terminal voltage equation is given by, $V = E_g - I_a R_a - I_a R_{se} - V_{br}$

Power developed in the DC generator = $E_g I_a$

Power delivered to the load = $V I_L$

Short shunt DC Compound generator

In short shunt DC compound generator, the shunt field winding is connected across the armature conductors and this combination is connected in series with a series field winding. The following figure shows the circuit diagram of short shunt DC compound generator.



In the above circuit diagram,

I_{sh} – Shunt field current, I_{se} – Series field current, I_a – Armature current, I_L – Load current,

R_a – Armature winding resistance, V – terminal voltage, V_{br} – Brush contact drop